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Article

Simplified Weight Function for Calculating Stress Intensity Factor in Complicated Stress Distributions

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Abstract. Calculation of a stress intensity factor becomes more difficult when crack are subjected to a complicated stress distribution profile. A standard procedure called influence coefficients is inadequate because a stress profile may not be accurately represented by a polynomial function. This paper applies a piecewise linear approximation of stress profile and a weight function method to overcome that restriction. However, the typically adopted weight function, i.e. universal weight function, is replaced by a weight function, in which its form coincides with the analytical form. Although a new weight function consists of lesser number of terms, it is proved to be accurate when applies to a cracked-cylinder problem, e.g. internal part-through circumferential crack and internal fully circumferential crack under various complicated weld residual stress profiles. Using this simpler weight function and linearized approximation scheme led to a closed-form stress intensity factor solution, which is convenient for programming.

Keywords: Weight function, stress intensity factor, fracture mechanics.

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1. Introduction

Industrial components such as pressure vessels or pipelines possibly contain a crack-like flaw that originated from a fabrication, e.g. welding or initiated and growth during service. A cracked component has to be assessed for their integrity and remaining useful life, i.e. perform a fitness-for-service (FFS) assessment. Several international recommended practice are developed for conducting FFS assessment, for example, API 579 [1], SINTAP/FITNET [2] etc. A review of the existing FFS recommended practices can be found in literature [3, 4].

One important step in the process of assessment is to determine a crack driving force, namely a stress intensity factor, *K*. The annex C of API 579 standard [5] provides *K*-solutions of many cracked components. This standard also adopts a method called influence coefficients for dealing with the case that crack surface is subjected to a complicated stress distribution, e.g. due to weld residual stress or thermal stress. To apply the influence coefficients method, the stress distribution along the component's wall thickness has to be fitted with the 4th order polynomial function. Note that the stress distribution may be determined from analytical, experimental or computational methods. But, the 4th order polynomial function sometimes inaccurately describes a stress distribution profile, especially in the case of a weld residual stress [6]. In this situation, a more general approach for computing *K* such as a weight function method is necessary. Because, it supports any functional form of stress distribution profile. However, the weight function has a singularity at the crack tip. As a result, special numerical and analytical techniques were proposed to minimize an error from the integration of a weight function [7, 8]. However, Wu [9] avoided the effect of singularity by approximating the stress distribution profile with a piecewise straight line. This approach results in the closed-form of *K*-solution, which is written as a summation of *K* contributed by each interval of linearized stress. This approach was further applied to many cracked bodies by Glinka [7] and successfully applied to several cases of weld residual stress [10]. Later, Shim [11] extended this approach by using a cubic spline interpolation instead of a linear interpolation and applied to a weld residual stress field. His results conform well to the finite element results.

A stress profile between discrete values of stress can be approximated by several approaches. Anderson [12] studied three approximation methods: piecewise constant, piecewise linear and piecewise quadratic. It is concluded that even the quadratic method is the most accurate, but it is difficult to implement when the stress changes abruptly. Thus, he recommended a piecewise linear representation of a stress profile. Note that, application of quadratic method does exist in the literature, for example, a work on a cracked cylinder under thermal transient load by Navabi et al. [13].

Recently, Li et al. [14] proposed a method called segment-wise polynomial interpolation. This method divides the stress profile into several segments. Each segment contained many discrete values of stress that can be fit by the 4th order polynomial function. The number of terms in weight function is also increased from typically 4 terms to 6 terms for a better accuracy. Even though this technique yields accurate results as compared with the finite element analysis (FEA) results; it requires attention from an analyst in a selection of the appropriate bounds for each segment. As a result, it is difficult to implement in a computer program.

Note that API 579 and the above researchers except Wu [9] used the 4-terms weight function in the form that was proposed by Glinka [15] which is usually known as universal weight function (UWF). However, the functional form of a universal weight function does not conform to that of derived from the analytical solution of a displacement field. The present author used to compare the UWF of a semi-infinite plate with edge crack [15] with the most accurate one [16] and found that the UWF deviated about 9%. Whereas, theoretical-form weight function deviated only 1.3% even it has only 3 terms. Results of a cylinder with an internal circumferential crack [17] also showed that if the coefficients in theoretical-form weight function were determined from the same reference cases as UWF, then UWF was less accurate. Therefore, it is interest to further explore the applicability of the simpler, i.e. 3-terms theoretical form weight function.

This paper focusses on the application of 3-terms theoretical form weight function and piecewise linear approximation of stress profile to compute *K*. First, a new closed-form *K*-solution is derived. Next, this derived equation is applied to a cylinder with an internal part-through circumferential crack and internal fully circumferential crack under complicated stress distributions. Finally, the computed *K* is compared with the finite element results from the literature [6, 11, 14] to evaluate its accuracy.

2. Theoretical Background

Weight function method for calculating a stress intensity factor, K was firstly proposed by Bueckner in 1970 and further discussed by Rice [18]. This method is very attractive because weight function is a property of a cracked body. Once the weight function for a cracked body is known, it can be used to find the K-solution of that cracked body subjected to arbitrary stress distribution profile. In the case of pure mode I (opening mode), the mode I stress intensity factor K_I of 2-dimensional cracked body having an edge crack (Fig. 1) is computed from

$$K_{\mathrm{I}} = \int_0^a \sigma(x) \cdot m_{\mathrm{I}}(x,a)dx \tag{1}$$

and

$$m_{\mathrm{I}} = \frac{E'}{K_{\mathrm{I}}} \frac{\partial v}{\partial a} \tag{2}$$

where m_{I} is mode I weight function.

$\sigma(x)$ is normal stress acting along the crack plane.

x is reference coordinate axis that is parallel to the crack plane.

a is crack length (or depth in the case of semi-elliptical surface crack, see Fig. 2).

v is a component of crack face displacement normal to a crack plane.

E' is an apparent Young's modulus, $E' = E$ for plane stress and $E' = E/(1-\nu^2)$ for plane strain. Note that ν is Poisson's ratio.

To find a weight function of a specific cracked body, a straight-forward method uses one reference loading case of that cracked body, which solutions of K and crack face displacement, v are known. By substituting these solutions into Eq. (2), the weight function m_{I} can be derived. This weight function can be used for calculating K_{I} of that crack body subjected to arbitrary stress distribution on the crack plane. However, this is not a practical way because the crack face displacement solution is rarely available as compared with a K-solution.

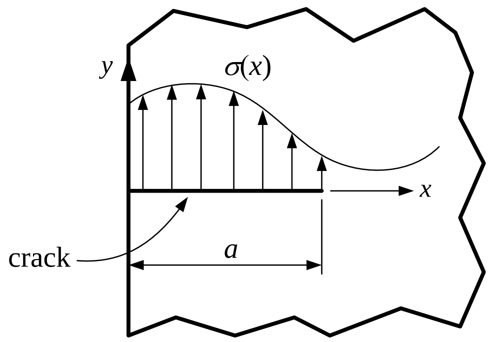


Fig. 1. Two-dimensional body with edge crack subjected to arbitrary stress distribution on the crack plane.

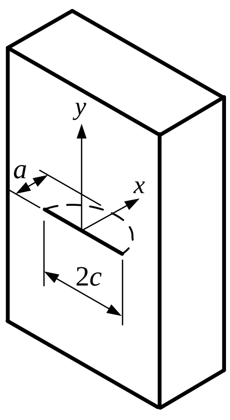


Fig. 2. Plate with semi-elliptical surface crack.

Petroski and Achenbach [19] proposed a semi-inverse method to overcome this restriction. They assumed a crack face displacement function that is truncated from an analytical solution of edge crack problem. Their function is written as

$$v(x,a) = C_1 \cdot a^{\frac{1}{2}}(a-x)^{\frac{1}{2}} + C_2 \cdot a^{-\frac{1}{2}}(a-x)^{\frac{3}{2}} \tag{3}$$

where the coefficients C_1 and C_2 are undetermined coefficients, which are a function of crack length.

After the substitution of Eq. (3) into (2), the derived weight function can be rearranged as the following form.

$$m_I(x,a) = \frac{2}{\sqrt{2\pi a(1-\rho)}}\left[1 + M_1(1-\rho) + M_2(1-\rho)^2\right] \tag{4}$$

where $\rho = x/a$

Petroski and Achenbach (PA) method requires one reference loading case that K-solution is known. This K-solution and Eq. (3) are then used to set up equations from two conditions called near-tip condition and self-consistency condition. Then, these equations are solved for the unknown coefficients M_1 and M_2. Note that PA method was successfully applied to many problems [20, 21, 22].

It should be noted that Eq. (4) is applicable to the deepest point (point A) of a semi-elliptical surface crack (Fig. 2). For convenient, it will be rewritten as:

$$m_A(x,a) = \frac{2}{\sqrt{2\pi a(1-\rho)}}\left[1 + M_1(1-\rho) + M_2(1-\rho)^2\right] \tag{5}$$

For a surface point (point B), a pure mode I weight function is expressed as follows:

$$m_B(x,a) = \frac{2}{\sqrt{\pi a\rho}}\left(1 + N_1\rho + N_2\rho^2\right) \tag{6}$$

Later, Shen and Glinka [23] proposed a universal weight function for the deepest and surface points of semi-elliptical surface crack in the following form:

$$m_A(x,a) = \frac{2}{\sqrt{2\pi a(1-\rho)}}\left[1 + M_1'(1-\rho)^{\frac{1}{2}} + M_2'(1-\rho) + M_3'(1-\rho)^{\frac{3}{2}}\right] \tag{7}$$

and

$$m_B(x,a) = \frac{2}{\sqrt{\pi a\rho}}\left(1 + N_1'\rho^{\frac{1}{2}} + N_2'\rho + N_3'\rho^{\frac{3}{2}}\right), \tag{8}$$

respectively.

They also proposed a method for determining the unknown coefficients, i.e. M', N', in Eq. (7) and Eq. (8) from 2 reference loading cases and a geometrical property of the crack face profile [24]. This method is later classified as a direct adjustment method (DAM).

Note again that if the unknown coefficients in Eq. (5) and Eq. (6) are determined by DAM with 2 reference loading cases, the accuracy of the weight function in Eq. (5) is improved [17] and becomes more accurate than the universal weight function.

3. K-solution by the Proposed Weight Function

This section presents a detailed derivation of K-solution under a piecewise linear approximation of the stress profile based on the weight functions in Eqs. (5) and (6). Weight function coefficients are determined from two reference loading cases, i.e. DAM. The first case is uniform stress distribution over the component wall thickness as shown in Fig. 3(a) and the second case is linearly distribution (Fig. 3(b)). The K-solutions of both loading cases are taken from the annex C of API 579 [3].

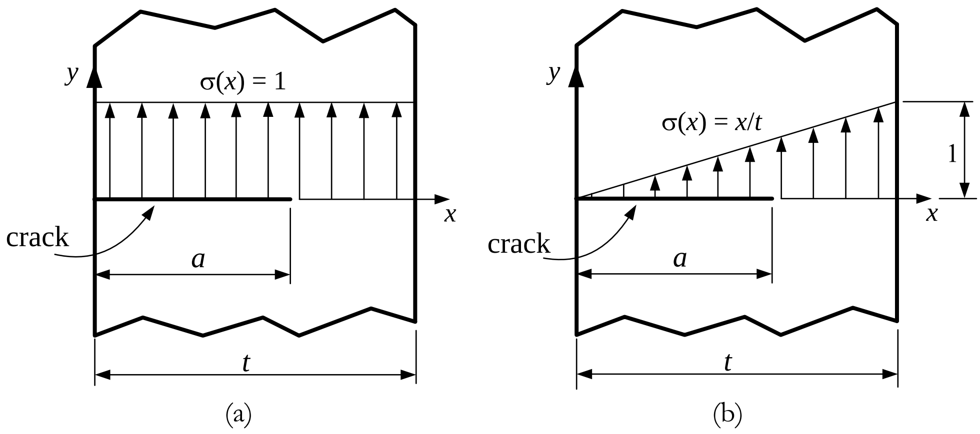


Fig. 3. Two references loading cases used for determining weight function coefficients: (a) Uniform stress distribution, (b) Linear stress distribution.

3.1. K-solution of the Reference Loading Cases

In API 579 [3], *K*-solution for a part-through circumferential crack (Fig. 4.) under uniform and linearly stress distribution over a wall thickness are

$$K_{\mathrm{I},uniform} = G_0\sqrt{\frac{\pi a}{Q}} \tag{9}$$

and

$$K_{\mathrm{I},linear} = G_1 \cdot \frac{a}{t} \cdot \sqrt{\frac{\pi a}{Q}}\ , \tag{10}$$

respectively.

where a, c are crack depth and half crack length, respectively.

$$Q = \begin{cases} 1+1.464(a/c)^{1.65} & ;a/c \le 1 \\ 1+1.464(c/a)^{1.65} & ;a/c > 1 \end{cases}$$

G_0, G_1 are influence coefficients that are calculated from the following equations

$$G_0 = \sum_{i=0}^{6} A_{i,0}\beta^i \tag{11}$$

$$G_1 = \sum_{i=0}^{6} A_{i,1}\beta^i \tag{12}$$

$$\beta = \frac{2\varphi}{\pi} \tag{13}$$

The elliptic angle φ at the surface and deepest points are zero and $\pi/2$, respectively. Note that, Q is unity for a fully circumferential crack (Fig. 5). The coefficients $A_{i,0}$ and $A_{i,1}$ for internal part-through and internal fully circumferential cracks, are listed in the Table C.14 and Table C.11 in API 579 [3], respectively. Note that these coefficients are a function of R_i/t, a/c and a/t.

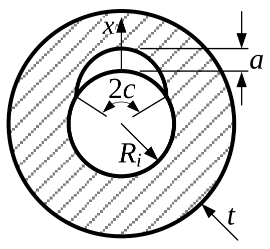


Fig. 4. A cylinder with internal part-through circumferential crack.

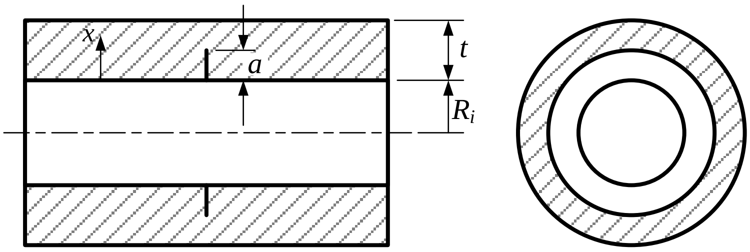


Fig. 5. A cylinder with internal fully circumferential crack.

3.2. Weight Function Coefficients

3.2.1. The deepest point

Substituting the first reference loading case, i.e. $\sigma(x) = 1$ and Eq. (5) into Eq. (1) yields

$$K_{\mathrm{I},uniform,A} = \int_0^a 1 \cdot \sqrt{\frac{2}{\pi a(1-\rho)}}\left[1 + M_1(1-\rho) + M_2(1-\rho)^2\right]dx$$

$$= \sqrt{\frac{2a}{\pi}} \cdot \int_0^1 \frac{1}{\sqrt{1-\rho}}\left[1 + M_1(1-\rho) + M_2(1-\rho)^2\right]d\rho$$

Integration of the equation yields

$$K_{\mathrm{I},uniform,A} = \frac{2}{15}\left(5M_1 + 3M_2 + 15\right)\sqrt{\frac{2a}{\pi}} \tag{14}$$

Substituting the second reference loading case, i.e. $\sigma(x) = x/t$ and Eq. (5) into Eq. (1) yields

$$K_{\mathrm{I},linear,A} = \int_0^a \frac{x}{t} \cdot \sqrt{\frac{2}{\pi a(1-\rho)}}\left[1 + M_1(1-\rho) + M_2(1-\rho)^2\right]dx$$

$$= \frac{a}{t} \cdot \sqrt{\frac{2a}{\pi}} \cdot \int_0^1 \frac{\rho}{\sqrt{1-\rho}}\left[1 + M_1(1-\rho) + M_2(1-\rho)^2\right]d\rho$$

$$K_{\mathrm{I},linear,A} = \frac{4}{105}\left(7M_1 + 3M_2 + 35\right)\sqrt{\frac{2a}{\pi}} \cdot \frac{a}{t} \tag{15}$$

Equate Eq. (14) to Eq. (9) and Eq. (15) to Eq. (10), and solve for the weight function coefficients. The result is

$$M_1 = -\frac{15\pi}{16}\sqrt{\frac{2}{Q}} \cdot \left(2G_0 - 7G_1\right) - 10 \tag{16}$$

and

$$M_2 = \frac{35\pi}{16}\sqrt{\frac{2}{Q}} \cdot \left(2G_0 - 5G_1\right) + \frac{35}{3} \tag{17}$$

3.2.2. The surface point

Substituting the first reference loading case, i.e. $\sigma(x) = 1$ and weight function in Eq. (5) into Eq. (1) yields

$$K_{\mathrm{I},uniform,B} = \int_0^a 1 \cdot \sqrt{\frac{2}{\pi a \rho}}\left(1 + N_1\rho + N_2\rho^2\right)dx$$

Integration of the equation yields

$$K_{\mathrm{I},uniform,B} = \frac{4}{15}\left(5N_1 + 3N_2 + 15\right)\sqrt{\frac{a}{\pi}} \tag{18}$$

Similarly, for the linearly stress distribution, $\sigma(x) = x/t$, we obtain

$$K_{\mathrm{I},linear,B} = \frac{4}{105}\left(21N_1 + 15N_2 + 35\right)\sqrt{\frac{a}{\pi}} \cdot \frac{a}{t} \tag{19}$$

Equate Eq. (18) to Eq. (9) and Eq. (19) to Eq. (10), and solve for the weight function coefficients. The result is

$$N_1 = \frac{15}{16}\frac{\pi}{\sqrt{Q}}\left(5G_0 - 7G_1\right) - 10 \tag{20}$$

and

$$N_2 = \frac{35}{16}\frac{\pi}{\sqrt{Q}}\left(5G_1 - 3G_0\right) + \frac{35}{3} \tag{21}$$

3.3. Piecewise Linear Approximation of K-solution

Figure 6 illustrates a piecewise linear approximation of a given discrete value of stress distribution along the thickness direction. The normal stress over *i*-th interval is approximated by:

$$\sigma_i = A_i x_i + B_i \tag{22}$$

The fitting coefficients are

$$A_i = \frac{\sigma_i - \sigma_{i-1}}{x_i - x_{i-1}} \tag{23}$$

$$B_i = \sigma_i - A x_i \tag{24}$$

where $i = 1, 2, 3, \ldots$

The required K-solution is the summation of K contributes from each interval. Therefore,

$$K_{\mathrm{I}} = \sum_{i=1}^{m} (K_{\mathrm{I}})_i \, dx \tag{25}$$

$$(K_{\mathrm{I}})_i = \int_{x_{i-1}}^{x_i} \sigma_i \cdot m_{\mathrm{I}}(x,a)\, dx \tag{26}$$

where m is the number of intervals over the crack length.

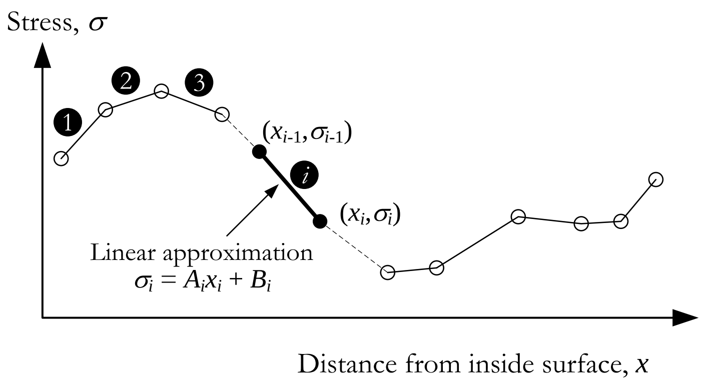


Fig. 6. Approximation of stress between two discrete points by a straight line.

For the deepest point, $(K_{\mathrm{I}})_i$ is determined using the weight function in Eq. (5) and its coefficients in Eq. (16) and (17). The closed-form solution of an integration above is denoted as $(K_{\mathrm{I},A})_i$ and is expressed as:

$$(K_{\mathrm{I},A})_i = \frac{2}{105}\sqrt{\frac{2a}{\pi}}\left[\left(7I_1A_ia + 35I_2B_i\right)M_1 - \left(3I_3A_ia + 21I_4B_i\right)M_2 - \left(35I_5A_ia + 105I_6B_i\right)\right] \tag{27}$$

$$I_1 = 3\left[(1-\rho_i)^{\frac{5}{2}} - (1-\rho_{i-1})^{\frac{5}{2}}\right] - 5\left[(1-\rho_i)^{\frac{3}{2}} - (1-\rho_{i-1})^{\frac{3}{2}}\right] \tag{28}$$

$$I_2 = \left[\rho_i(1-\rho_i)^{\frac{1}{2}} - \rho_{i-1}(1-\rho_{i-1})^{\frac{1}{2}}\right] - \left[(1-\rho_i)^{\frac{1}{2}} - (1-\rho_{i-1})^{\frac{1}{2}}\right] \tag{29}$$

$$I_3 = 2\left[(1-\rho_i)^{\frac{5}{2}} - (1-\rho_{i-1})^{\frac{5}{2}}\right] + 5\left[\rho_i(1-\rho_i)^{\frac{5}{2}} - \rho_{i-1}(1-\rho_{i-1})^{\frac{5}{2}}\right] \tag{30}$$

$$I_4 = \left[(1-\rho_i)^{\frac{1}{2}} - (1-\rho_{i-1})^{\frac{1}{2}}\right] \cdot \left[(\rho_i - \rho_{i-1})^2 - (1-\rho_i)^{\frac{1}{2}}(1-\rho_{i-1})^{\frac{1}{2}}(\rho_i + \rho_{i-1} - 2) + 3(\rho_i - 1)(\rho_{i-1} - 1)\right] \tag{31}$$

$$I_5 = 2\left[(1-\rho_i)^{\frac{1}{2}} - (1-\rho_{i-1})^{\frac{1}{2}}\right] + \left[\rho_i(1-\rho_i)^{\frac{1}{2}} - \rho_{i-1}(1-\rho_{i-1})^{\frac{1}{2}}\right] \tag{32}$$

$$I_6 = (1-\rho_i)^{\frac{1}{2}} - (1-\rho_{i-1})^{\frac{1}{2}} \tag{33}$$

For the surface point, $(K_I)_i$ is determined using the weight function in Eq. (6) and its coefficients in Eq. (20) and (21). The closed-form solution of an integration which is denoted as $(K_{I,B})_i$ is expressed as:

$$(K_{I,B})_i = \frac{4}{105}\sqrt{\frac{a}{\pi}}\left[\left(21J_1A_ia + 35J_2B_i\right)N_1 + \left(15J_3A_ia + 21J_4B_i\right)N_2 + \left(35J_5A_ia + 105J_6B_i\right)\right] \tag{34}$$

$$J_1 = J_4 = \rho_i^{\frac{5}{2}} - \rho_{i-1}^{\frac{5}{2}} \tag{35}$$

$$J_2 = J_5 = \rho_i^{\frac{3}{2}} - \rho_{i-1}^{\frac{3}{2}} \tag{36}$$

$$J_3 = \rho_i^{\frac{7}{2}} - \rho_{i-1}^{\frac{7}{2}} \tag{37}$$

$$J_6 = \rho_i^{\frac{1}{2}} - \rho_{i-1}^{\frac{1}{2}} \tag{38}$$

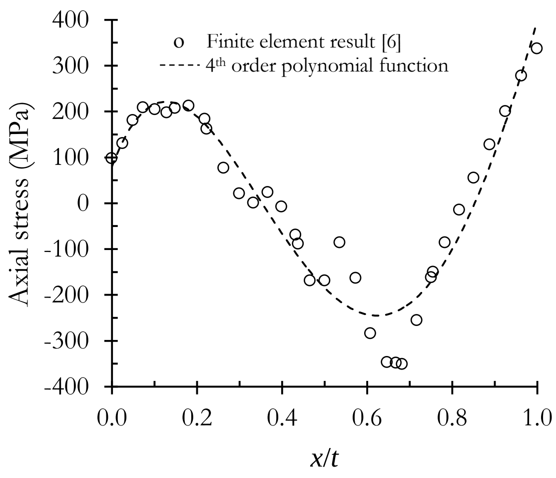
4. Validation

This section summary the validation results of derived *K*-solution under a complicated weld residual stress profile taken from the literature [6, 11, 14]. These profiles are illustrated in Fig. 7 and are denoted as profile number 1, 2, 3, 4 and 5. Note that an individual value of stresses and positions along the thickness direction are digitized from a figure in the literature. It can be seen that the best-fit 4th order polynomial function fails to reproduce the shape of the stress profiles no. 2, 3 and 5. All validation cases are summarized in Table 1.

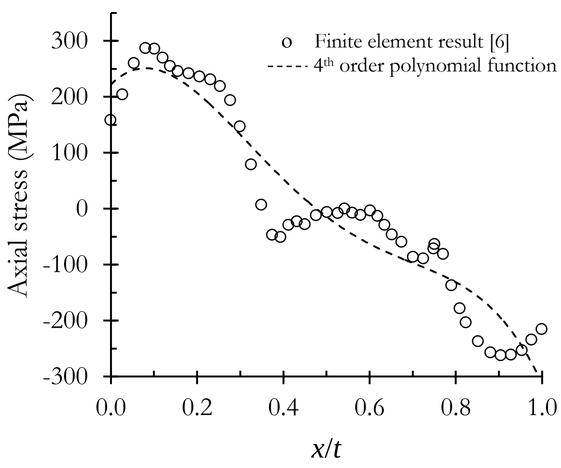
Table 1. Validation cases.

Cases	Crack	*t (mm)*	R_i/t	*a/c*	Profile no.	Location	Reference
1	Part-through circumferential	40	5	1	1, 2	Deepest	[6]
2	Part-through circumferential	40	5	1/3	1, 2	Deepest	[6]
3	Part-through circumferential	47.33	3	1/5	3	Deepest & Surface	[11]
4	Fully circumferential	40	5	-	1, 2	Deepest	[6]
5	Fully circumferential	30	60	-	4, 5	Deepest	[14]

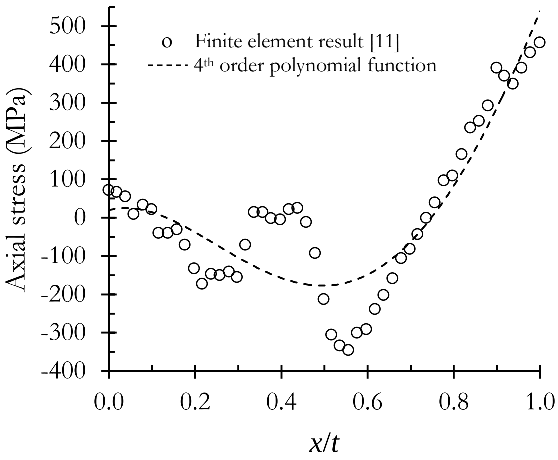
To perform a validation; first, the slope and intercept of a line joining a discrete value of a stress profile were calculated from Eqs. (23) and (24). Next, determine the influence coefficients, i.e. G_0, G_1 from the appropriate tables in API 579. Note that G_0, G_1 depend on the ratio of crack depth to a wall thickness, i.e. a/t, then an interpolation is necessary for a/t values that are not tabulated. After, influence coefficients are known the weight function coefficients can be determined from Eqs. (16) and (17) for the deepest point or Eqs. (20) and (21) for the surface point. Finally, the K_I for all intervals are computed from Eq. (25), together with the Eqs. (27) to (33) for the deepest point or Eqs. (34) to (38) for the surface point.



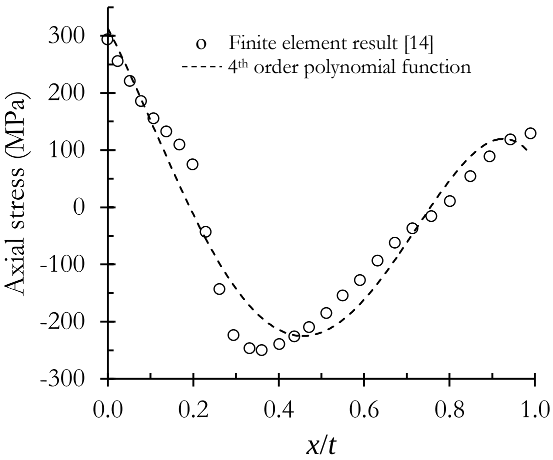
(a) Profile no. 1



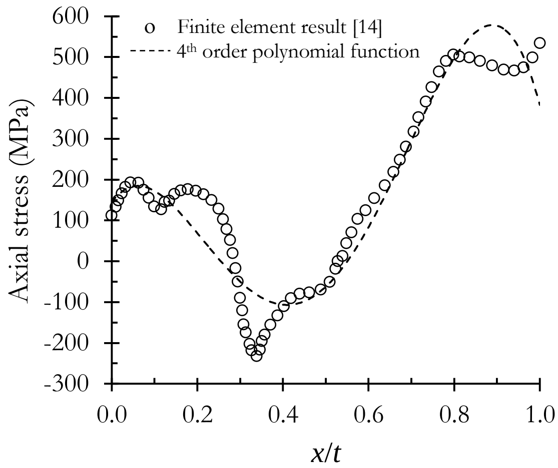
(b) Profile no. 2



(c) Profile no. 3



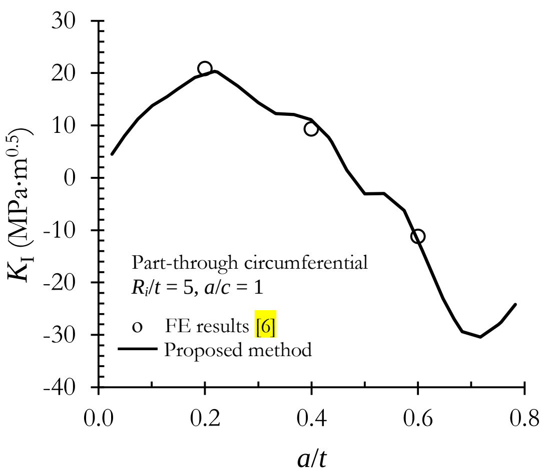
(d) Profile no. 4



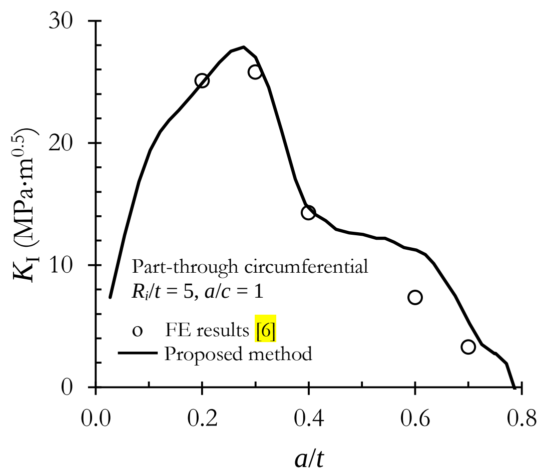
(e) Profile no. 5

Fig. 7. Weld residual stress profiles used in validation of a proposed weight function and a piecewise linear approximation of stress profile.

The results of the cases 1 to 5 are shown in the Figs. 8 to 12, respectively. The results are limited to the cases with $a/t \leq 0.8$, because of the applicability of the influence coefficients in API 579. A plot of the FE results is also shown for comparison. It can be seen that the calculation results by the proposed weight function conforms well to the FE results.



(a) Profile 1, Deepest point



(b) Profile 2, Deepest point

Fig. 8. Comparison of K_I estimated by piecewise linear approximation with FE solutions for case no. 1.

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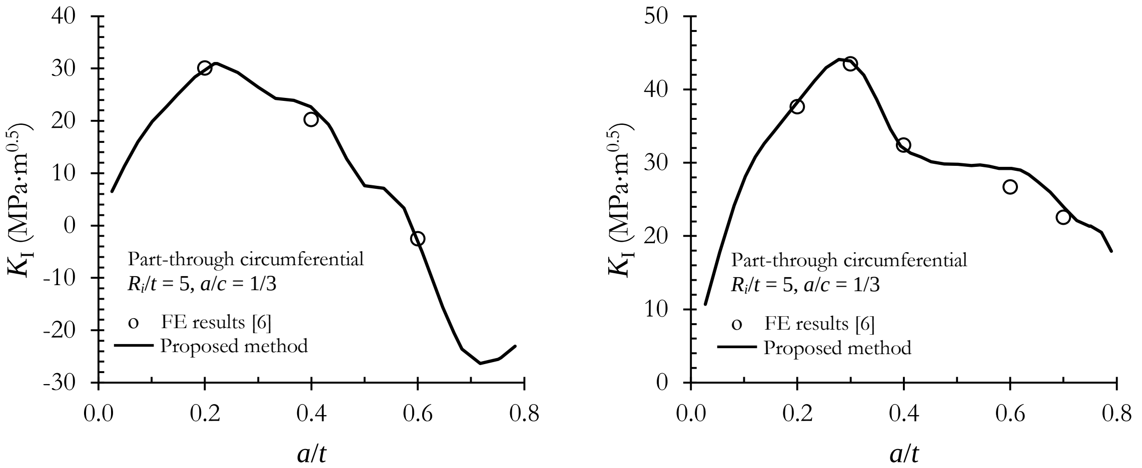
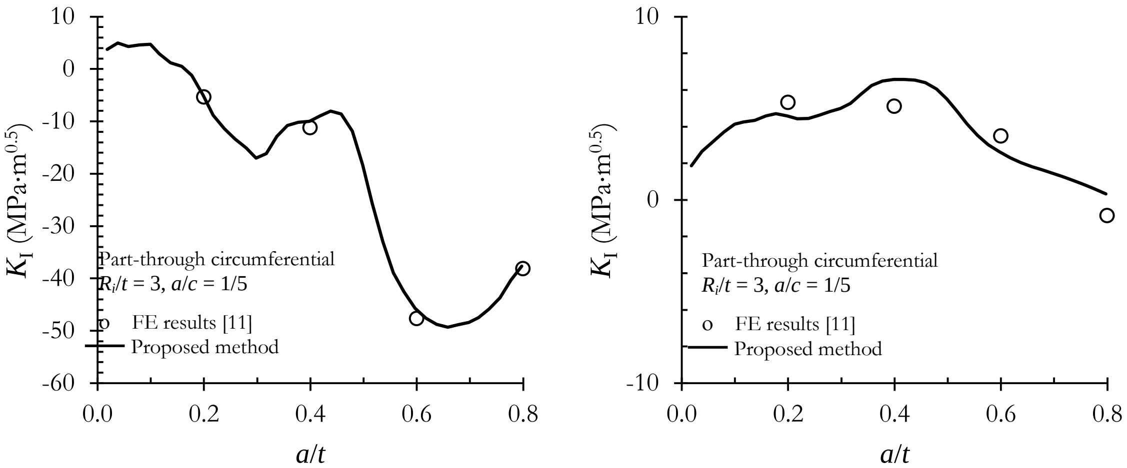


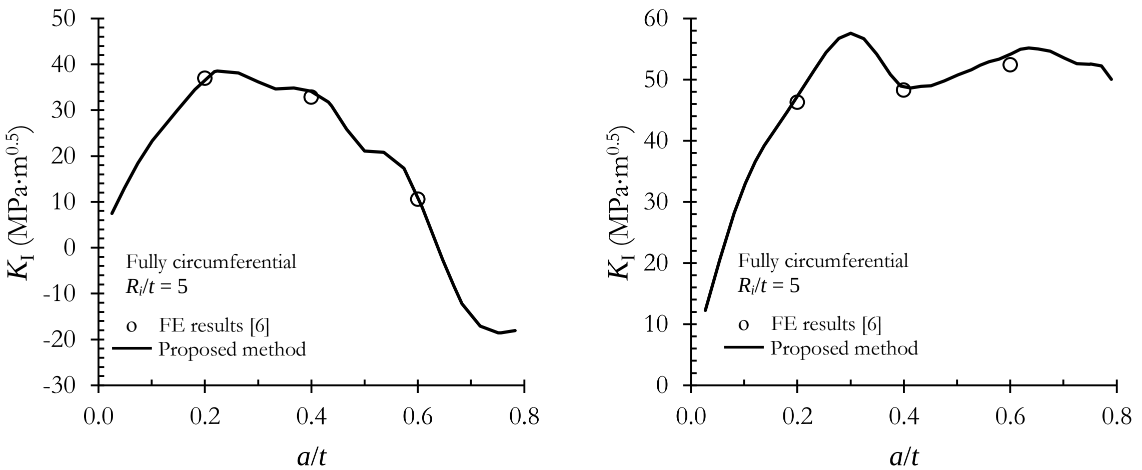
(a) Profile 1, Deepest point (b) Profile 2, Deepest point

Fig. 9. Comparison of K_I estimated by piecewise linear approximation with FE solutions for case no. 2.



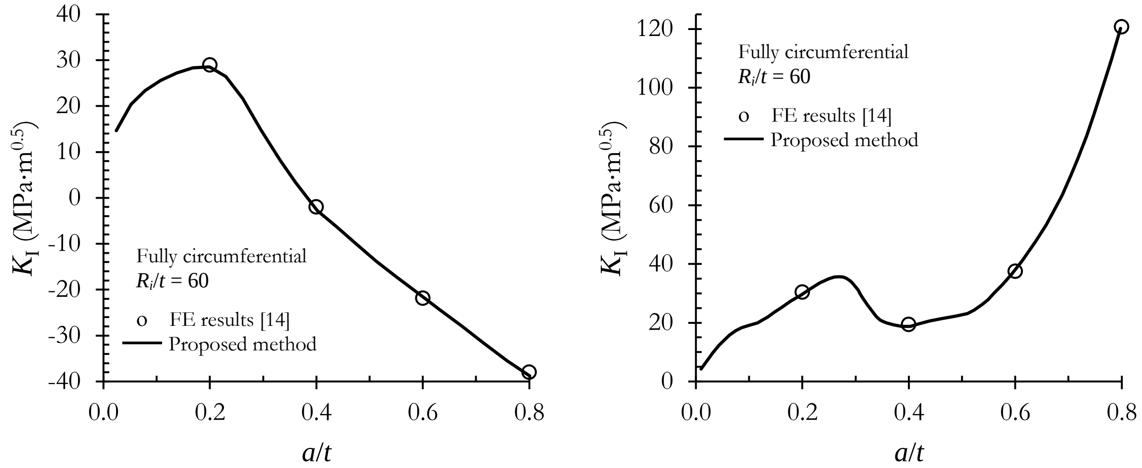
(a) Profile 3, Deepest point (b) Profile 3, Surface point

Fig. 10. Comparison of K_I estimated by piecewise linear approximation with FE solutions for case no. 3.



(a) Profile 1, Deepest point (b) Profile 2, Deepest point

Fig. 11. Comparison of K_I estimated by piecewise linear approximation with FE solutions for case no. 4.



(a) Profile 3, Deepest point (b) Profile 4, Deepest point

Fig. 12. Comparison of K_I estimated by piecewise linear approximation with FE solutions for case no. 5.

5. Conclusions

This study proposed a method for calculating K using a simpler weight function, i.e. a 3-terms theoretical form weight function, and a piecewise linear approximation of stress profile. Weight function coefficients were determined from two reference loading cases available in API 579 standard. This weight function was used to derive a new closed-form K-solution. The derived solution was validated by computing a K_I at the deepest and surface points of an internal part-through circumferential crack and the deepest point of an internal fully circumferential crack under various complicated weld residual stress profiles. It was shown that the proposed method could accurately calculate K in comparison with FE results and easy to program.

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