doi:10.4186/ej.2010.14.2.33

MULTIDIMENSIONAL DISSIPATION TECHNIQUE FOR ROE'S FLUX-DIFFERENCE SPLITTING

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ABSTRACT

A multidimensional dissipation technique is implemented on the Roe's flux difference splitting scheme to avoid the numerical shock instability that may occur in compressible flow solutions. The damping characteristic of the proposed technique is presented through a linear perturbation analysis on the problem of a moving shock along an odd-even grid perturbation. The performance of the proposed technique is studied using well-known problems that exhibit the numerical shock instability. The technique is further extended to achieve higher-order solution accuracy and evaluated by several benchmark test cases.

KEYWORDS

shock instability, carbuncle phenomenon, Roe's FDS, entropy fix, *H*-correction

doi:10.4186/ej.2010.14.2.33

I . Introduction

Numerical flux formulation is an essential part of flux formulation schemes in order to obtain accurate and robustness numerical solutions of the Euler equations. The Roe's Flux-Difference Splitting (FDS) scheme [1] is widely used by the CFD community; however, it may provide unrealistic flow solutions or lead to numerical instability in certain problems. These problems include the carbuncle phenomenon [2] that refers to a spurious bump on the bow shock near the flow center line ahead the blunt body; an unrealistic perturbation [3] that occurs from a moving shock along odd-even grid perturbation in a straight duct; and a kinked Mach stem observed when a normal shock wave reflects on a ramp to form a double-Mach reflection. To improve the solution accuracy of these problems, Quirk [3] pointed out that the original Roe's FDS should be modified in the vicinity of strong shock.

The objectives of this paper are to propose a multidimensional dissipation technique for the Roe's FDS scheme on triangular meshes and to evaluate the technique by using several benchmark test cases. The entropy fix method by Van Leer *et al*. [4], and the *H*-correction entropy fix method by Pandolfi *et al*. [5] are modified for unstructured triangular meshes and implemented into the Roe's FDS scheme. The performance of these methods [4]-[6] is evaluated by well-known problems that exhibit the numerical shock instability. The proposed multidimensional dissipation technique for the Roe's FDS scheme is described. The problem of a moving shock along an odd-even grid perturbation in a straight duct is used to demonstrate the dissipation mechanism that relates to the numerical instability. Finally, the higher-order extension of the Roe's FDS scheme is implemented to the flow solution accuracy.

II. Numerical Shock Instability

In this section, the Roe's FDS and the Roe's FDS with the three entropy fix methods [4]-[6] are presented and examined on four examples to demonstrate their numerical instability or unrealistic solution behaviors. All solutions in this section are obtained using the first-order accuracy on structured triangular meshes.

2.1 Roe's FDS with dissipation

The governing differential equations of the Euler equations for the two-dimensional inviscid flow are given by,

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{E}}{\partial x} + \frac{\partial \boldsymbol{G}}{\partial y} = 0 \tag{1}$$

where $\boldsymbol{U}$ is the vector of conservation variables, $\boldsymbol{E}$ and $\boldsymbol{G}$ are the vectors of the convection fluxes in x and y directions, respectively. For perfect gas, the equation of state is in the form $p = \rho e(\gamma - 1)$ where p is the pressure, ρ is the density, e is the internal energy, and γ is the specific heat ratio.

The numerical flux vector at the cell interface between the left cell L and the right cell R according to the Roe's FDS (**Roe**) [1] is,

$$\boldsymbol{F}_n = \frac{1}{2}(\boldsymbol{F}_{nL} + \boldsymbol{F}_{nR}) - \frac{1}{2}\sum_{k=1}^{4} \alpha_k |\lambda_k| \boldsymbol{r}_k \tag{2}$$

where α_k is the wave strength of the k^{th} wave, λ_k is the eigenvalue, and $\boldsymbol{r}_k$ is the corresponding right eigenvector.

The Van Leer's entropy fix method (**RoeVL**) is designed to correct the unphysical expansion shock originated by **Roe**. The one-dimensional entropy fix was developed by replacing the characteristic speeds of the acoustic waves (for k = 1 and 4) with,

$$|\lambda_k|^* = \begin{cases} |\lambda_k| & , |\lambda_k| \ge 2\eta^{VL} \\ \dfrac{|\lambda_k|^2}{4\eta^{VL}} + \eta^{VL} & , |\lambda_k| < 2\eta^{VL} \end{cases} \tag{3}$$

where $\eta^{VL} = \max(\lambda_R - \lambda_L, 0)$.

Sanders *et al*. [6] introduced an idea of a multidimensional dissipation, the so called *H*-correction entropy fix method (**RoeSA**). For the two triangular cells as shown in Figure. 1, the *H*-correction entropy fix has been modified to [7],

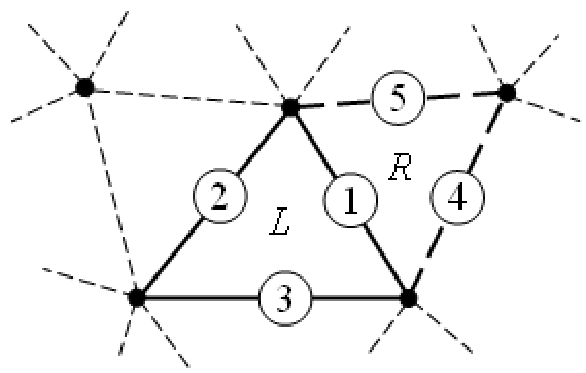
$$\eta^{SA}=\max(\eta_1,\eta_2,\eta_3,\eta_4,\eta_5) \quad (4)$$

where $\eta_i = 0.5\max_k(\left|\lambda_{i,kR} - \lambda_{i,kL}\right|)$.

Pandolfi *et al*. [5] proposed another version of the *H*-correction entropy fix by excluding the η_1 from Eq. (4) to avoid an erroneous injection of artificial viscosity, and is applicable only to the entropy and shear waves (for *k* = 2 and 3). The *H*-correction entropy fix modified by Pandolfi *et al*. (**RoePA**) is,

$$\eta^{PA} = \max(\eta_2,\eta_3,\eta_4,\eta_5) \quad (5)$$

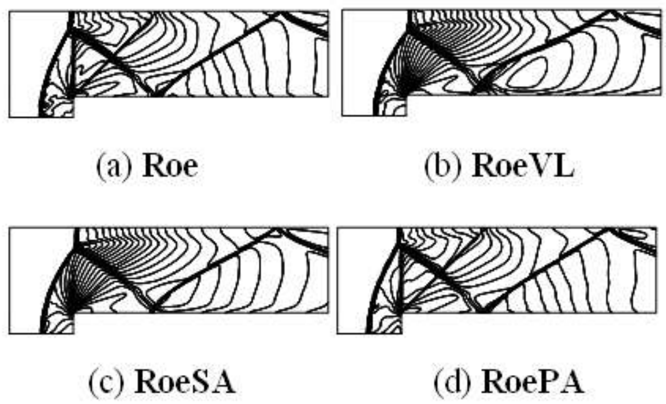
Figure 1
Cell interfaces of unstructured triangular grid.



2.2 The expansion shock

To illustrate an unphysical expansion shock, a Mach 3 flow over a forward facing step [8] is investigated. The density contours computed from the **Roe**, **RoePA**, **RoeSA**, and **RoeVL** are shown in Figures. 2(a)-(d), respectively. The figures show that **Roe** and **RoePA** produces an unphysical expansion shock on top of the facing step corner, whereas both the **RoeSA** and **RoeVL** provide reasonable solutions.

Figure 2
Mach 3 flow over a forward facing step.



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2.3 The kinked mach stem

The problem is described by a normal shock that reflects from a ramp to form a double-Mach reflection. Unrealistic numerical solution may consist of a kinked Mach stem and the flaw triple point. Figures 3(a)-(d) show that **Roe** and **RoeVL** yield severely kinked Mach stem [9] whereas **RoeSA** and **RoePA** provide reasonable accurate solutions.

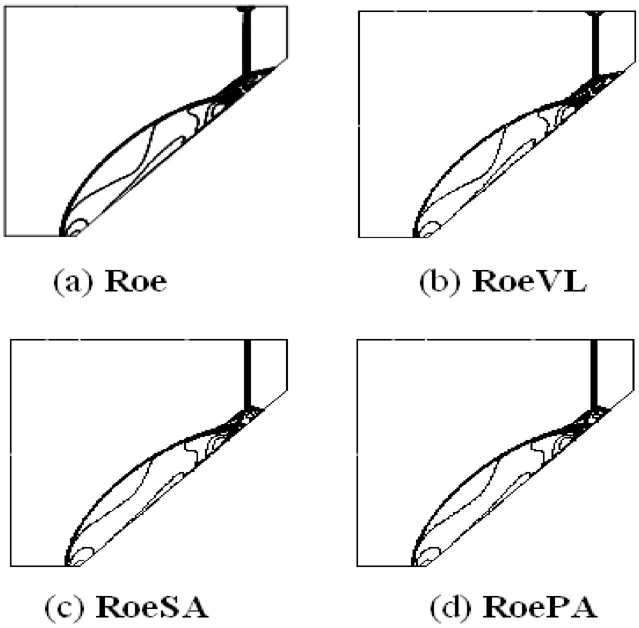

Figure 3
Mach 5 shock moving over a 46^o ramp.

III. Multidimensional Dissipation Technique for Roe's FDS Scheme

The flow behaviors obtained from the test cases in Section 2 using the **RoeVL**, **RoeSA** and **RoePA** schemes, which were modified to avoid the numerical shock instability, are examined. The detailed study suggests that a mixed entropy fix method (**RoeVLPA**) that combines the entropy fix method of Van Leer and the modified *H*-correction of Pandolfi should be used by replacing the original eigenvalues as,

$$|\lambda_k|^* = \begin{cases} |\lambda_{1,4}| & ,|\lambda_{1,4}| \geq 2\eta^{VL} \\ \dfrac{|\lambda_{1,4}|^2}{4\eta^{VL}} + \eta^{VL} & ,|\lambda_{1,4}| < 2\eta^{VL} \\ \sqrt{\kappa}\max\left(|\lambda_{2,3}|, 0.5\eta^{PA}\right) & \end{cases} \quad (6)$$

where η^{VL} and η^{PA} are defined in Eqs. (3) and (5), respectively. The constant value κ is usually less than or equal to one for the first-order accuracy scheme and more than one for higher-order solution accuracy (for simplicity the value of one is used throughout this paper, otherwise an explicit value is specified). The performance of the mixed entropy fix method is re-evaluated by solving the previous two test cases presented in Figures. (2)-(3) again. Figures (4)-(5) show that the proposed **RoeVLPA** can capture the shock accurately without any numerical instability. These figures show that the **RoeVLPA** is good enough for stabilizing the original Roes FDS scheme.

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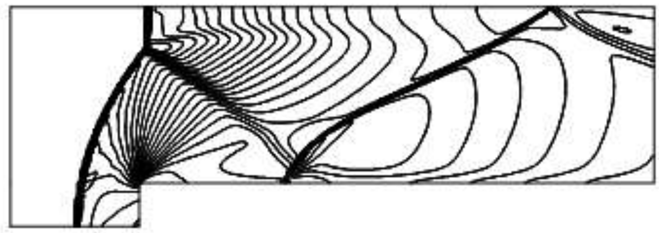

Figure 4
Mach 3 flow over a forward facing step.

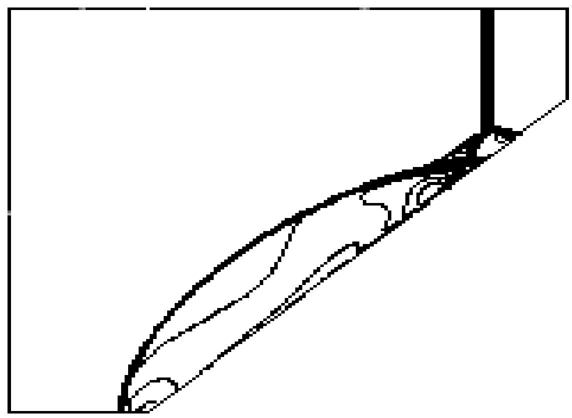

Figure 5
Mach 5 shock moving over a 46^o ramp.

IV. Second-Order Extension

Solution accuracy from the first-order formulation described in the preceding sections can be improved by implementing a high-order extension for both the space and time. High-order spatial discretization is achieved by applying the Taylor' series expansion to the cell-centered solution for each cell face [10] and Vekatakrishnan's limiter function [11] for preventing spurious oscillation that may occur in the region of high gradients. Second-order temporal accuracy is achieved by implementing the second-order accurate Runge-Kutta time stepping method [12]. The time step determination proposed by Linde and Roe [13] is used for all the analyses in this paper.

4.1 Symmetric rarefaction wave problem

The initial conditions of the symmetric rarefaction wave problem [13] are given by $(\rho, u, p)_L$ = (7.0, -1.0, 0.2) and $(\rho, u, p)_R$ = (7.0, 1.0, 0.2), such that they produce a vacuum at the center of domain. Figure 6 shows the higher-order solutions from the **RoeVLPA** that compare well with the exact solutions.

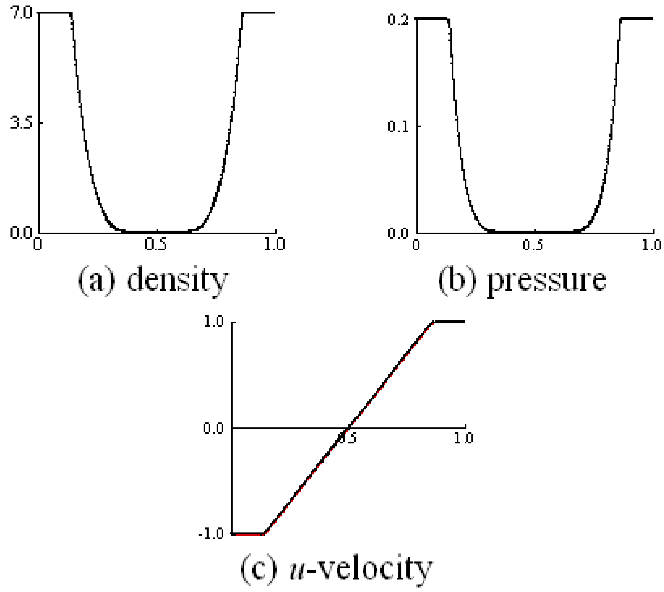

Figure 6
Comparative exact and numerical solutions at time t = 0.3 for the symmetric rarefaction wave problem.

4.2 Mach 2 shock reflection over a wedge

A Mach 2 shock reflection over a wedge at 46 degrees [14] is used to evaluate the performance of the proposed **RoeVLPA** as compared to the other schemes for a more complex flow problem. All numerical experiments were performed using a structured triangular mesh with 256 $\times$ 256 nodes. Figure 7 compares the density contours obtained from the four schemes at the time the shock wave is at distance of 0.1 from the right boundary. Using the constant κ = 1 in the **RoeVLPA** scheme, the incident shock is slightly broken-down with severely kinked Mach stem due to an inappropriate numerical dissipation added to the vorticity and entropy waves as shown in Figure. 7(a). The solution from the **RoeVLPA** is improved with good shock and Mach stem resolution after using κ = 6 as shown in Figure. 7(b). Figures 7(c) and (d) show the more dissipative solutions from the AUSM-PW+ [15] and Zha & Bilgen FVS [16] schemes that yield good flow resolution with little spurious triple point and slight kinked Mach stem near the wall.

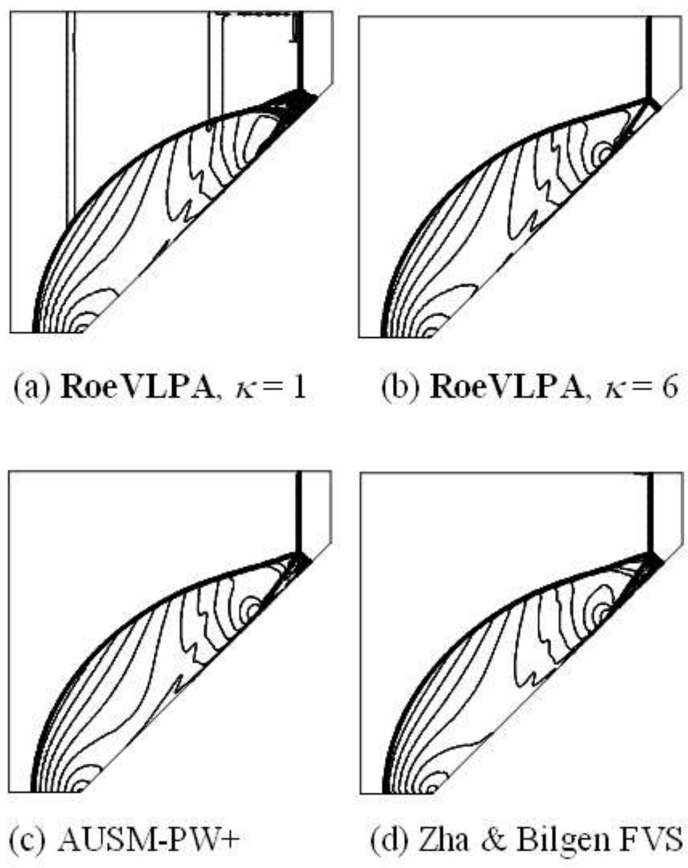

Figure 7
Comparison of density contours of a Mach 2 shock reflection over a wedge.

V. Conclusions

A multidimensional dissipation technique is presented to improve numerical stability of the Roe's FDS scheme. The performance of the method was evaluated by several well-known test cases and found to eliminate unphysical solutions that may arise from the use of the original Roe's FDS scheme. The high-order spatial and second-order Runge-Kutta temporal discretization was implemented to further improve the solution accuracy. Such implementation was found that the **RoeVLPA** scheme provides accurate solutions while avoiding the numerical shock instability.

ACKNOWLEDGMENT

The author is pleased to acknowledge the Thailand Research Fund (TRF) for supporting this research work.

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