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Article

Constrained Joint Replenishment Problem with Refrigerated Vehicles

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Abstract. We study a constrained joint replenishment problem with a multi-commodity refrigerated road transport in cold chain logistics. Each truck may have multiple temperature zones, since products in full truckload shipment may have different temperature requirements. In the proposed mathematical programming model, we want to minimize the expected total cost that includes the inventory cost and the transportation cost as well as the penalty cost if temperature violation occurs subject to the full truckload constraint. Under the deterministic demand, the cycle time of each product, the temperature of each zone in each truck and the allocation plan (the number of units of each product to be shipped in each zone in each truck) are obtained from the mixed-integer nonlinear optimization model. Under the stochastic demand, we assume that the inventory is controlled using a periodic review system, and the order-up-to level is chosen to maintain the desired cycle service level of each product. In the case study of one of the largest modern grocery retailers in Thailand, our model is applied to obtain the optimal replenishment policy. Currently, the company's fleet consists of single-temperature trucks. We estimate the monetary benefit obtained by switching from a single-temperature truck to a multi-temperature truck. We also estimate the cost reduction from reducing the lead time. Finally, our model can be used to quantify the trade-off between the service level and the inventory cost to help the company choose the appropriate service levels.

Keywords: Applied operations research, cold chain logistics, joint replenishment problem, multi-commodity multi-temperature refrigerated transport.

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1. Introduction

Cold chain logistics is crucial to temperature-sensitive products such as perishable products (e.g., fresh fruits, vegetables, meats) and medical products (e.g., vaccines, blood and plasma). The cold chain logistics market worldwide is expected to exceed 585 billion USDs in five years [1]. The cold chain consists of at least four linked systems, namely precooling, warehouse refrigeration, refrigerated transport and marketing [2]. Since different products have different temperature requirements, cold storage categories include high-temperature (5 to 15 °C), medium-temperature (−5 to 5 °C), low-temperature (−25 to −18 °C), quick-freezing (−40 to −30 °C) and ultralow-temperature (−60 to −45 °C) refrigerated warehouses. Rather than reducing the cargo temperature, the main objective of the refrigerated transport is to maintain a stable and uniform temperature during the delivery period. In this article, we focus on road transport from a distribution center to retailers via refrigerated vehicles. We optimize the coordinated ordering decision of different products and the shipping plan, which specify both volume and temperature point in each temperature zone in each truck.

The literature review is as follows. The joint replenishment problem (JRP) is surveyed in [3] and [4], in which the latter considers the JRP under stochastic demand. The basic JRP is extended to include various resource constraints such as transportation capacities, warehouse capacities and budget limitations. The JRP with resource constraints is sometimes referred to as the constrained joint replenishment problem (CJRP). In [5], the CJRP has a single budget constraint, and the model is solved using the genetic algorithm. [6] solves the CJPR with a single constraint using a heuristic solution from linear programming. Our CJRP model has more than one constraint, stating that the total allocation to each zone in each truck cannot exceed its capacity. The capacity constraints are included in [7] and [8]. The study of [9] considers the CJRP with stochastic demand under the periodic-review inventory system, in which the dynamic order-up-to levels are used to create full truckloads. In [9], the CJRP with full truckloads is solved in two steps: The first step determines the number of trucks, and the second step determines the dynamic order-up-to levels in such a way that the total shipped volume leads to a full truckload. Unlike [9], we solve for the shipping and ordering decisions simultaneously in one unified CJRP model. [10] formulates the JRP as a mixed-integer linear programming problem, in which the setup (ordering) cost is stepwise and depends on the number of trucks used. In [10], the planning horizon is finite, whereas in ours the planning horizon is infinite. In [10], the demand is deterministic and nonstationary, whereas in ours the demand is stochastic and stationary.

Our model also has a stepwise ordering cost, which is a function of the number of trucks used. Furthermore, we consider refrigerated trucks and include the temperature penalty cost if the truck temperature is outside the designated temperature range of each product. In our model, the transport capacity is a hard constraint as in the previous studies. Nevertheless, in ours the temperature range is a soft constraint and a violation penalty is included in the objective function. In the previous deterministic JRP model, the average total cost includes the average setup cost and the average holding cost. In ours, the average total cost includes both the setup and holding costs as well as the penalty cost associated with temperature violation during refrigerated transport.

A comprehensive review of cold chain logistics can be found in [2] for fresh agricultural products and in [11] for food products. [12] studies the temperature management problem in refrigerated warehouse, and the optimal target temperature is obtained for each multi-commodity cold storage room in the warehouse. [12] considers the temperature management for warehouse refrigeration, whereas we consider that for refrigerated transport. [13] studies the cold chain management problem in the hierarchical hub network. Ours is a single echelon, but we focus on the refrigerated transportation.

For perishable product, the JRP can be found in [14] and [15], and their models account for non-instantaneous deterioration and random lifetime, respectively. [16] formulates a JRP for products with varying ages. [17] develops an inventory policy with micro-periodic control, which allows delivery of a perishable product several times daily. [18] compares different inventory policies for perishable goods. [19] considers the multi-echelon inventory optimization in the meat supply chain. [20] studies the special case of the multi-echelon inventory system with one warehouse and multiple retailers, and [21] considers a periodic-review inventory policy in the two-echelon inventory problem with seasonal demand. The pricing decision for perishable products is studied in [22], [23], [24] and [25]. [26] studies both pricing and replenishment decisions with expiration date dependent deterioration. [27] studies both pricing and preservation technology investment decision. These studies determine the prices of different products so that the revenue is maximized, whereas in ours, we determine the different temperatures of zones in trucks so that the cost is minimized. Table 1 summarizes the key attributes of the above literature. These studies on the JRP for perishable products do not explicitly account for both the temperature control inside the truck and the truck capacity, which are two key features in our model.

Our key contribution is formulating the CJRP model with refrigerated delivery in a full truckload multi-temperature refrigerated truck. Unlike the traditional mono-temp (single-temp) truck, this so-called multi-temp truck allows different temperatures in different zones within one truck. The customized refrigeration sections (zones) can be created using the thermal insulated barriers called bulkheads; see Fig. 1. From Table 1, none of the previous studies with temperature control considers this multi-temp truck. In [13], the refreshing and freezing operations are done in a hub node, and items are transported in delivery vehicles with ambient temperature. [24] introduces the cold-chain service level to implicitly capture cold-chain operations such as packing, cooling facilities and temperature level during transportation. The temperature point in the refrigerated vehicle is not modeled explicitly. In [27], [25] and [12], cold storage at warehouses or retailers are considered. Our model determines the temperature inside each zone in each multi-temp truck. Since the mono-temp truck is a special case of ours, our model can be used to evaluate the monetary benefit from using the multi-temp truck.

We formulate the CJRP with refrigerated delivery. Many products with different temperature targets are transported in full truckload. In spite of the multi-temp truck, the temperature inside each zone in each truck may still exceed the upper bound of the target temperature of one product, while falling below the lower bound of another product. As in the goal programming approach, when the zone temperature falls outside the designated bound of each product, a penalty cost from temperature violation is incurred. Our objective is to minimize the average total cost. In our deterministic model, the average total cost includes the average setup cost that depends on the number of trucks used, the average holding cost that depends on the cycle stock, and the penalty cost from temperature violation. In our stochastic model, the expected holding cost also depends on the safety stock. Our stochastic CJRP is a mixed-integer nonlinear optimization with a chance constraint to ensure that the in-stock probability of each product is at least its desired service level.

The rest of this paper is organized as follows: In Section 2, we propose the CJRP with the temperature control management and extend to account for demand uncertainty. We identify conditions under which the problem becomes convex programming in Section 3. A case study is provided in Section 4. Concluding remarks and a few extensions are given in Section 5.

2. Formulation

Consider a joint replenishment problem with a family of n coordinated items, called items $i = 1, 2, \ldots, n$. These items can be delivered using m trucks. Let L be the lead time, which is assumed to be constant. The lead time is the period of time from which an order is placed until it is received. The lead time includes the order preparation time, the transit time from a supplier and time for inspection after receiving the order. The input parameters are given in the top part of Table 2. There are three types of setup costs, namely the full truckload (FTL) cost of truck j denoted as K_j, the minor setup cost of item i denoted as k_i, and the other fixed major setup cost K_o. The FTL cost K_j may depend on the truck size; for instance, the 18-wheel truck may have a larger FTL cost than the 6-wheel truck. Similarly, the maximum and minimum temperature points, y_j^L and y_j^U, may be different for each truck. The cost K_j also includes the pre-trip cost associated with pre-cooling trucks, which may take upto a few hours. The minor setup cost of each item, k_i, may include the procurement cost such as the cost of order approval and receipt, the billing cost and the cost of incoming inspection. In practice, the capacity κ_j is slightly below the total truck volume, since the total truck volume may be sufficient, but the shipments may not fit inside the truck due to their different shapes. This is sometimes referred to as stacking loss. In practice, the capacity κ_j is approximately 80 percent of the total truck volume.

Throughout this article, let $\mathbb{N}$ denote the set of natural number, and let $(x)^+ = \max(x, 0)$ denote the positive part of a real number x.

2.1. Deterministic Demand

Assume that demand is deterministic. Let d_i be the demand rate of item i. Recall the standard JRP, we want to determine the family cycle time or the base cycle T and the cycle time of each item, so that the average total cost is minimized. The cycle time of item i, the time interval between two consecutive replenishment times of item i, is $T_i = m_i T$ where $m_i \in \mathbb{N}$. Under the assumption that demand is deterministic, determining the cycle time T_i is equivalent to determining the order quantity q_i since $q_i = d_i T_i$ for each item i. The cycle time T_i is the time supply of item i, since the replenishment quantity, q_i, will last for T_i time units. The family cycle time is the time intervals between replenishments of the family, and item i will be included in every m_ith replenishment of the family. Also under the deterministic demand assumption, the zero inventory ordering property holds: Every order is received precisely when an inventory position drops to zero. If lead time is zero, then an order is placed when a stock level becomes zero.

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Table 1. Key attributes of related literature (1="Yes"; 0="No").

Article	Multiprod.	Temp. Contr.	Aging & Deteriorat.	Stoch. Demand	Truck Cap.	Pricing
[22]	1	0	0	1	0	1
[23]	1	0	1	0	0	1
[13]	1	1	0	1	0	0
[19]	1	0	1	1	0	0
[24]	0	1	1	0	0	1
[18]	1	0	1	1	0	0
[25]	1	1	1	0	0	1
[26]	0	0	1	0	0	1
[27]	0	1	1	0	0	1
[28]	1	0	0	1	0	0
[7]	1	0	1	0	1	0
[17]	0	0	1	1	0	0
[8]	1	0	1	0	1	0
[29]	1	0	1	1	0	0
[14]	1	0	1	0	0	1
[15]	1	0	1	1	0	0
[30]	1	0	0	0	0	1
[12]	1	1	0	0	0	0
[16]	0	0	1	0	1	0
[9]	1	0	0	0	1	0
Ours	1	1	0	1	1	0

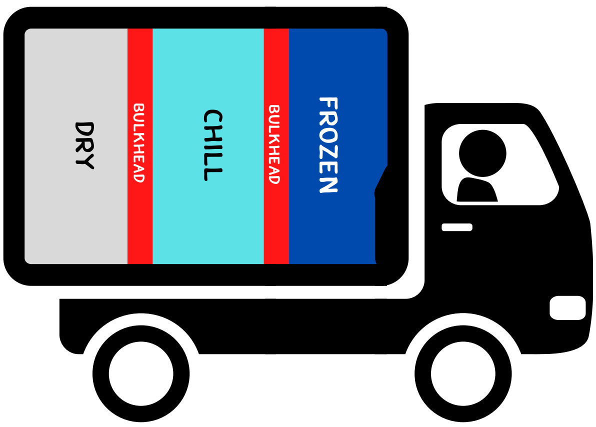

Fig. 1. Insulated bulkheads in multi-temp truck.

Table 2. Input parameters and decision variables.

Input parameter	
n	$=$ number of items
m	$=$ number of trucks
p	$=$ number of temperature zones in each truck
d_i	$=$ demand rate of item i (unit/time unit)
h_i	$=$ holding cost of item i (monetary unit/unit/time unit)
k_i	$=$ minor setup cost of item i (monetary unit/delivery)
K_j	$=$ FTL cost of truck j (monetary unit/delivery)
K_o	$=$ other fixed major setup cost (monetary unit/delivery)
t_i^U	$=$ upper bound of item i's target temperature (°C)
t_i^L	$=$ lower bound of item i's target temperature (°C)
p_i^U	$=$ penalty cost from exceeding upper bound t_i^U (monetary unit/°C/unit)
p_i^L	$=$ penalty cost from falling below lower bound t_i^L (monetary unit/°C/unit)
v_i	$=$ volume of item i (cubic meter/unit)
κ_j	$=$ capacity of truck j (cubic meter)
y_j^L	$=$ minimum temperature point on truck j (°C)
y_j^U	$=$ maximum temperature point on truck j (°C)
L	$=$ lead time (time unit)
Decision variable	
T	$=$ base cycle or family cycle time (time unit)
m_i	$=$ integer number of T intervals
T_i	$=$ cycle time of item i (time unit)
q_i	$=$ order quantity of item i (unit)
x_{ijk}	$=$ allocation quantity of item i on truck j in zone k (unit)
y_{jk}	$=$ temperature point of truck j in zone k (°C)
z_{jk}	$=$ planned volume of zone k in truck j (cubic meter)
δ_{ijk}^{U+}	$=$ positive deviation from upper bound t_i^U of item i on truck j in zone k (°C)
δ_{ijk}^{U-}	$=$ negative deviation from upper bound t_i^U of item i on truck j in zone k (°C)
δ_{ijk}^{L+}	$=$ positive deviation from lower bound t_i^L of item i on truck j in zone k (°C)
δ_{ijk}^{L-}	$=$ negative deviation from lower bound t_i^L of item i on truck j in zone k (°C)
u_j	$=$ binary variable equal to 1 if truck j is used and 0 otherwise

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If lead time is positive, then an order of size q_i is placed, when the inventory position of item i is less than or equal to d_iL, the reorder point of item i.

We extend the standard JRP to include the shipping plan and the temperature control inside each zone in each truck. We consider a multi-temperature refrigerated truck, and the current practice of using a single-temperature refrigerated truck is a special case of ours. In the standard JRP, the average total cost includes the average setup cost and the average holding cost, whereas in ours, the average penalty cost from temperature violation is included as well. The temperature violation occurs if the zone temperature falls below the lower bound of the target temperature of each item, or if it exceeds the upper bound of the target temperature of each item. All decision variables are given in the bottom part of Table 2. The mathematical programming model is given as follows:

$$\begin{aligned}\text{Min } & \frac{1}{T}(K_o + \sum_{j=1}^{m} u_j K_j) \\ & + \sum_{i=1}^{n} \Big(\frac{k_i}{T_i} + \frac{h_i q_i}{2} \Big) \\ & + \sum_{i=1}^{n} \Big(\frac{1}{T_i} \sum_{j=1}^{m} \sum_{k=1}^{p} (p_i^U \delta_{ijk}^{U+} + p_i^L \delta_{ijk}^{L-}) x_{ijk} \Big) \quad (1)\end{aligned}$$

subject to:

$$\begin{aligned} T_i &= m_i T, & \forall i \quad (2) \\ q_i &= d_i T_i, & \forall i \quad (3) \\ y_{jk} - \delta_{ijk}^{U+} + \delta_{ijk}^{U-} &= t_i^U, & \forall i,j,k \quad (4) \\ y_{jk} - \delta_{ijk}^{L+} + \delta_{ijk}^{L-} &= t_i^L, & \forall i,j,k \quad (5) \\ \sum_{i=1}^{n} v_i x_{ijk} &\le z_{jk}, & \forall j,k \quad (6) \\ \sum_{k=1}^{p} z_{jk} &\le \kappa_j, & \forall j \quad (7) \\ \sum_{j=1}^{m} \sum_{k=1}^{p} x_{ijk} &= q_i, & \forall i \quad (8) \\ \sum_{k=1}^{p} z_{jk} &\le \kappa_j u_j & \forall j \quad (9) \\ m_i &\in \mathbb{N}, & \forall i \quad (10) \\ u_j &\in \{0,1\}, & \forall j \\ x_{ijk}, \delta_{ijk}^{U+}, \delta_{ijk}^{U-}, \delta_{ijk}^{L+}, \delta_{ijk}^{L-} &\ge 0, & \forall i,j,k \\ y_j^L \le y_{jk} &\le y_j^U, & \forall j,k \end{aligned}$$

In the objective function (1), the first term is the average major setup cost, and the second term is the average minor setup cost and the average holding cost. In the third term, the second summation is the penalty cost from violating the temperature target of item i on all trucks in all zones; we multiply this by the average order frequency of item i ($1/T_i$) to obtain the average total penalty cost per time unit. The cycle time of item i is given in (2), where m_i is the integer number of T intervals that the replenishment quantity of item i, q_i, will last. The order quantity is equal to the demand rate times the cycle time; see (3). Constraints (4)–(5) together with the non-negativity constraint are used to define the deviational variables. Note that δ_{ijk}^{L+} (resp., δ_{ijk}^{U-}) does not appear in the objective function, since we do not get penalized from being over the lower bound t_i^L (resp., under the upper bound t_i^U). Constraint (6) states that the total volume across all items in zone j in truck k is at most its planned volume z_{jk}. The truck capacity constraint is given in (7). Constraint (8) states that the sum of units of item i in all trucks and all zones must be equal to the replenishment quantity q_i. In (9), the binary u_j must be 1 if truck j is used. Since item i will be included in every m_ith replenishment of the family, m_i needs to be a natural number as in (10).

In fact, the objective function in (1) is the upper bound on the average total cost, since we assume that the truck temperature remains the same for all replenishment cycles. If $m_i = 1$ for all $i = 1, 2, \ldots, n$, then all items are replenished in every cycle, and our objective function becomes an exact expression. In general, at the optimal, the value of m_i may be different for each item i, so the cycle time m_iT needs not be the same for all items. In reality, the truck temperature could be adjusted dynamically for each replenishment cycle, and the penalty cost would be lower than that in (1). However, the exact penalty cost has no closed-form expression. Our model uses the upper bound as an approximation, served as the abstraction of the reality.

We assume that all trucks have identical pre-specified routes. The distribution center or a regional warehouse serves a fixed set of nearby retailers, each of which demands all n items. (The demand rate d_i is the sum of all demand rates from all retailers.) Each truck may delivery different items, but all trucks must deliver to all retailers, so that each retailer receives all n items. All retailers locate in one region. Each truck originates from the distribution center, visits all retailers according to a pre-specified route, and returns back to the distribution center. The routing decision can be obtained from solving a Chinese postman problem, and it is beyond the scope of this paper. The general vehicle routing problem is studied in, e.g., [31], [32] and [33]. In this paper, we determine how to allocate demand on each item in each temperature zone in each truck. Under the assumption of pre-specified route, the total travel distance of each truck becomes constant. The transportation cost associated with the total travel of each truck is included in the fixed major setup cost K_j. This is found in previous literature such as [9] and [8].

2.2. Stochastic Demand

Assume that demand follows a continuous distribution, so that the cumulative distribution function (CDF) is strictly increasing, and the inverse of the CDF is well defined. Recall that for deterministic demand, the order quantity for item i is constant among all cycles and equal to $q_i = d_i T_i$, where $T_i = m_i T$ is the cycle time of item i. For stochastic demand, the order quantity for item i needs not be equal among all cycles, and an order-up-to level (OUTL), denoted S_i is used to determine the order quantity. If the inventory position of item i is IP_i, then the order quantity is $(S_i - \mathrm{IP}_i)^+$. The order quantity of item i is equal to the random demand during one cycle, denoted as D_i^r. In analogous to (3), the expected order quantity of item i is

$$\bar{Q}_i = \mathrm{E}[D_i^r] = d_i T_i.$$

In the steady state (equilibrium), the inflow during one cycle must be equal to the outflow during one cycle. The inflow is the expected order quantity and the outflow is the expected demand during one cycle.

We consider the periodic review system with control parameters (T_i, S_i), where the inventory position is reviewed every T_i time unit. This time interval T_i is called the review period, which is the time that elapses between consecutive moments at which we know the inventory position. Note that if the demand during every review period is strictly positive, then an order is placed at every review period, and the cycle time and the review period are identical. If during a particular review interval, there is no demand, then no order is placed at the review period, and the cycle time could be longer than the review period. We assume that the demand distribution is chosen such that the probability of the latter (no demand during the review period) is negligible; this is often found in practice. The protection period, i.e., the interval of time over which a stock-out is possible, is equal to the sum of the lead time and review period. For instance, for item i if an inventory position is reviewed every month (4 weeks), and it takes two weeks from which an order is placed until it is received, then the review period is $T_i = 4$, and the lead time $L = 2$; thus, the protection period is $T_i + L = 6$, given that one time unit is one week. The effective demand D_i^e is the demand during the production period, $T_i + L$. The OUTL S_i is determined such that the in-stock probability is at least the desired cycle service level α_i; i.e., $\Pr(D_i^e \leq S_i) \geq \alpha_i$. In each cycle, the expected on-hand inventory is

$$\bar{I}_i = \frac{\bar{Q}_i}{2} + \mathrm{E}[(S_i - D_i^e)^+],$$

where $\mathrm{E}[(S_i - D_i^e)^+]$ is the expected on-hand inventory at the end of the cycle, and $\bar{Q}_i/2 = \mathrm{E}[D_i^r]/2$ is the average cycle stock. Let $F_i(\cdot;\ell)$ be the CDF of demand during ℓ time units. The demand in one cycle D_i^r has the distribution $F_i(\cdot;T_i)$, and the effective demand D_i^e has the distribution $F_i(\cdot;T_i+L)$. In each cycle, the expected number of backorders is

$$\bar{B}_i = \mathrm{E}[(D_i^e - S_i)^+] = \int_{S_i}^{\infty} \bar{F}_i(t;T_i+L)dt.$$

Note that the expected on-hand inventory at the end of the cycle can be expressed as

$$\begin{aligned}\mathrm{E}[(S_i - D_i^e)^+] &= (S_i - \mathrm{E}[D_i^e]) + \mathrm{E}[(D_i^e - S_i)^+] \\ &= \mathrm{SS}_i + \bar{B}_i\end{aligned}$$

where the safety stock (SS) is defined as the expected inventory level at the end of cycle.

Additional input parameters are

α_i = desired cycle service level of item i

γ_i = desired probability that the truck allocation for item i is at least its random order quantity

b_i = backorder cost of item i (monetary unit/unit/time unit)

g_i = shipping penalty cost of item i if its order quantity exceeds the truck allocation (monetary unit/unit)

The additional decision variable is

$$S_i = \text{OUTL of item } i \text{ (unit)}.$$

The mathematical programming model is given as follows:

$$\mathrm{Min}\ \frac{1}{T}(K_o + \sum_{j=1}^{m} u_j K_j) \quad (11)$$

$$+ \sum_{i=1}^{n} (\frac{1}{T_i}(k_i + \sum_{j=1}^{m}\sum_{k=1}^{p}(p_i^U \delta_{ijk}^{U+} + p_i^L \delta_{ijk}^{L-})x_{ijk}))$$

$$+ \sum_{i=1}^{n} \frac{g_i}{T_i} \mathrm{E}[(D_i^r - \sum_{j=1}^{m}\sum_{k=1}^{p} x_{ijk})^+] \quad (12)$$

$$+ \sum_{i=1}^{n} (h_i \bar{I}_i + b_i \bar{B}_i) \quad (13)$$

subject to:

$$T_i = m_i T, \qquad \forall i$$
$$y_{jk} - \delta^{U+}_{ijk} + \delta^{U-}_{ijk} = t^U_i, \qquad \forall i,j,k$$
$$y_{jk} - \delta^{L+}_{ijk} + \delta^{L-}_{ijk} = t^L_i, \qquad \forall i,j,k$$
$$\sum_{i=1}^{n} v_i x_{ijk} \le z_{jk}, \qquad \forall j,k$$
$$\Pr(D^r_i \le \sum_{j=1}^{m}\sum_{k=1}^{p} x_{ijk}) \ge \gamma_i, \qquad \forall i \tag{14}$$
$$\Pr(D^e_i \le S_i) \ge \alpha_i \qquad \forall i \tag{15}$$
$$\sum_{k=1}^{p} z_{jk} \le \kappa_j u_j \qquad \forall j$$
$$m_i \in \mathbb{N} \qquad \forall i$$
$$u_j \in \{0,1\} \qquad \forall j$$
$$x_{ijk}, \delta^{U+}_{ijk}, \delta^{U-}_{ijk}, \delta^{L+}_{ijk}, \delta^{L-}_{ijk} \ge 0, \qquad \forall i,j,k$$
$$y^L_j \le y_j \le y^U_j, \qquad \forall j$$

In the objective function, (12) is the expected penalty cost if the truck allocation cannot accommodate the entire order quantity, and (13) is the sum of the expected holding and backorder costs. Constraints (14) and (15) are chance constraints. In (14), for each item i, we are $100\gamma_i$ percent sure that the entire order quantity D^r_i does not exceed the truck allocation. In (15), the in-stock probability is at least α_i; i.e., we are $100\alpha_i$ percent sure that the stock-out does not occur. Let $F^{-1}_i(\cdot;\ell)$ be the inverse of the CDF $F_i(\cdot;\ell)$, i.e., the quantile of demand during ℓ time units. Then, the chance constraints (14)–(15) can be written as the following lower bounds:

$$\sum_{j=1}^{m}\sum_{k=1}^{p} x_{ijk} \ge F^{-1}_i(\gamma_i; T_i) \tag{16}$$
$$S_i \ge F^{-1}_i(\alpha_i; T_i + L). \tag{17}$$

Specifically, if we assume that the demand of item i in each time unit is independent and normally distributed with mean μ_i and standard deviation σ_i. The constraints (16)–(17) can be written as

$$\sum_{j=1}^{m}\sum_{k=1}^{p} x_{ijk} \ge \mu_i T_i + \Phi^{-1}(\gamma_i)\sigma_i\sqrt{T_i} \tag{18}$$
$$S_i \ge \mu_i(T_i + L) + \Phi^{-1}(\alpha_i)\sqrt{T_i + L}. \tag{19}$$

Furthermore, the expected cost in (12) can be computed using the standard normal loss function $L(z) = \mathrm{E}[(Z - z)^+] = \phi(z) - z(1 - \Phi(z))$. The expected order size that cannot be accommodated by the truck allocation is

$$\mathrm{E}[(D^r_i - \sum_{j=1}^{m}\sum_{k=1}^{p} x_{ijk})^+] = \sigma_i\sqrt{T}_i\, L\,(z)$$

where

$$z = \frac{\sum_{j=1}^{m}\sum_{k=1}^{p} x_{ijk} - \mu_i T_i}{\sigma_i\sqrt{T}_i}.$$

3. Analysis

In general, our model is a mix-integer nonlinear optimization problem. In this section, we will consider the special case with stochastic demand, where we fix the cycle time of each item T_i (i.e., m_i and T) and the temperature in each zone in each truck y_{jk}, and we optimize the OUTL S_i and the allocation plan x_{ijk}. This special case can be viewed as an initial improvement of the current situation, where we try to change only the OUTL and the allocation plan, leaving the other variables as currently stand. As a quick win, one can use a clustering algorithm to group items by their minimum and maximum temperature points. Items in the same cluster (group) have similar minimum and maximum temperature points to each other than those in other clusters. To employ a clustering algorithm, we need to specify the number of clusters, which should be at most the maximum number of temperature zones of all trucks. Then, the temperature in each zone in each truck y_{jk} could be a "central" temperature point among all items within the same cluster.

Proposition 1. *In the special case where m_i, T and y_{jk} are fixed, if the binary constraint is relaxed (i.e., $0 \le u_j \le 1$ for all j), then the associated continuous optimization is a convex programming problem.*

Proof. Suppose that m_i, T and y_{jk} are fixed for all i,j,k and that the binary constraint is relaxed, i.e., $0 \le u_j \le 1$ for all j. Then, the deviational variables $\delta^{U+}_{ijk}, \delta^{U-}_{ijk}, \delta^{L+}_{ijk}, \delta^{L-}_{ijk}$ are no longer decision variables but constants. The remaining decision variables are the OUTL S_i and the allocation quantity x_{ijk}.

For each item i, for shorthand notation, let $\mathbf{x}_i = (x_{ijk} : j = 1,\ldots,m,\ k = 1,\ldots,p)$. Note that

$$\mathrm{E}[(D^r_i - \sum_{j=1}^{m}\sum_{k=1}^{p} x_{ijk})^+] = f_i(g_i(\mathbf{x}_i))$$

where $f_i(z) = \mathrm{E}[(D^r - z)^+]$ and $g_i(\mathbf{x}_i) = \sum_{j=1}^{m}\sum_{k=1}^{p} x_{ijk}$. Clearly, $f_i(z)$ is convex and nonincreasing in z, since $(d - z)^+$ is convex and nonincreasing for all d, and $g_i(\mathbf{x}_i)$ is linear (both concave and convex). It follows from B-10 in [34] that $f_i(g_i(\mathbf{x}_i))$ is convex in $\mathbf{x}_i$. Since the nonnegative weighted sum of the convex functions is convex, the allocation penalty cost $\sum_{i=1}^{n} \frac{g_i}{T_i}\mathrm{E}[(D^r_i - \sum_{j=1}^{m}\sum_{k=1}^{p} x_{ijk})^+]$ given in (12) in the objective is convex in x_{ijk}.

Next, consider the sum of the expected holding and backorder costs $\sum_{i=1}^{n}(h_i\bar{I}_i + b_i\bar{B}_i)$ given in (13) in the objective. For any real numbers x and a, $(x - a) = (x - a)^+ - (a - x)^+$. Then,

$$\begin{aligned} & h_i \bar{I}_i + b_i \bar{B}_i \\ &= h_i(\frac{\bar{Q}_i}{2} + \mathrm{E}[(S_i - D_i^e)^+]) + b_i \,\mathrm{E}[(D_i^e - S_i)^+] \\ &= (h_i + b_i) \,\mathrm{E}[(D_i^e - S_i)^+] + h_i S_i + \tilde{c}_i \end{aligned}$$

where $\tilde{c}_i = h_i(\bar{Q}_i/2 - \mathrm{E}[D_i^e])$. Since $\mathrm{E}[(D_i^e - S_i)^+]$ is convex in S_i, the sum $\sum_{i=1}^{n}(h_i \bar{I}_i + b_i \bar{B}_i)$ is also convex in S_i.

We have shown that (12) is convex in x_{ijk} and (13) is convex in S_i. In the special case, the other terms in the objective are constant. Hence, the objective function is convex in S_i and x_{ijk}. Recall that the chance constraints correspond to the lower bounds given in (16) and (17) and that the other constraints are linear. We conclude that the special case with the binary relaxation is a convex programming problem. □

In the special case with the binary relaxation, Proposition 1 ensures convex optimization; thus, a local minimizer becomes the global minimizer. In the general case, convex programming cannot be guaranteed. For instance, if we fix m_i and T but determine y_{jk}, x_{ijk} and S_i, then the deviational variables δ_{ijk}^{L-} and δ_{ijk}^{U+} would become decision variables. Note that $\delta_{ijk}^{U+} x_{ijk}$ (or $\delta_{ijk}^{L-} x_{ijk}$) is quadratic but neither concave nor convex; we cannot ensure convexity of the temperature penalty cost term $\sum_{j=1}^{m} \sum_{k=1}^{p} (p_i^U \delta_{ijk}^{U+} + p_i^L \delta_{ijk}^{L-}) x_{ijk}$ in the objective function. In practice, different initial solutions should be tried multiple times. From our computational experiences in Section 4, the off-the-shelf optimization solver such as BARON can be used to solve a medium-sized problem within reasonable time (e.g., within an hour in all problem instances in Section 4).

Consider a large-scale problem where the number of items n may be hundreds or thousands. One feasible solution is the current policy that the company actually uses in practice, and we know the review period of each item T_i, the set of trucks used u_j, the temperature of each zone in each truck y_{jk} and the allocation plan x_{ijk} as well as the OUTL S_i. We can construct another feasible solution by optimizing only the allocation plan x_{ijk} and the OUTL S_i while keeping the other as is. We still have a large number of decision variables; specifically, if there are m trucks used, then the number of decision variables are $mnp + n = n(mp + 1)$. From Proposition 1, it is a convex programming problem, for which there are several available algorithms such as interior point, gradient projection and ellipsoid methods. The interior-point methods are reliable and "can solve problems with hundreds of variables and thousands of constraints on a current desktop computer, in at most a few tens of seconds" ([35], page 8). To solve our original problem, we could propose a simple heuristic approach, which consists of two procedures. In the inner-most procedure, we solve the convex program to find the allocation plan and the OUTL given the review period, the set of trucks used and the temperature. In the outer-most procedure, we determine these review period, the set of trucks used and the temperature using a metaheuristic algorithm, e.g., an evolutionary algorithm, tabu search and simulated annealing.

4. Case Study

In this case study, we consider a joint replenishment problem of one of the largest modern grocery retailers in Thailand. We consider $n = 18$ items sold in the retailers in the Bangkok metropolitan region. Their locations include Serithai, Ladprao 101, Kurngthep Kreetha, Keha Chalhongkrung, Soi Kamnan Yen-Uthid, On Nuch 80, Phaholyothin 52, Sangkha Santisuk, Talad Wongsakorn, Phetchakasem 114, Charan-Sanitwong 15 and Ramintra 67. These retailers are replenished from the distribution center located in the outskirt of Bangkok.

The input parameters for all items are given in Table 3. The highest demand item, raw meat (RWM $i = 14$), has the daily demand of 2397 units, and each unit requires 0.039 cubic meter, so the daily volume becomes 93.56 cubic meters. The minimum and maximum temperatures of the raw meat are -2 and 4 °C, respectively. The lowest demand item, chili paste (CHP $i = 2$), has the daily demand of 10 units, and each unit requires 0.0033 cubic meter, so the daily volume becomes 0.03 cubic meter. The minimum and maximum temperatures of the chili paste are 5 and 25 °C, respectively. Figure 2 shows the minimum and maximum temperatures of all items; the size of each point corresponds to the daily volume $d_i v_i$ (given in cubic meter/day) for each item.

The company has three 18-wheel (18W) trucks and three 4-wheel (4W) trucks for the delivery from the distribution center to these retailers. Let the number of trucks be $m = 6$. The three 18W trucks are called $j = 1, 2, 3$, and the three 4W trucks are called $j = 4, 5, 6$. For each truck type, the dimension and the corresponding volume as well as the FTL cost are shown in Table 4. When an order is placed, the company incurs the other fixed ordering cost of $K_o = 200$ THB. Assume that the penalty cost associated with exceeding the temperature upper bound $p_i^U = 0.5$ THB/°C/unit, and that with falling from the temperature lower bound $p_i^L = 0.5$ THB/°C/unit.

The current policy is as follows: All items are replenished at the same time; their cycle time is one day:

Table 3. Input parameters for deterministic JRP.

Item i	Code	Description	Min Temp (°C) t_i^L	Max Temp (°C) t_i^U	Daily Demand (unit/day) d_i	Unit Volume (m^3/unit) v_i	Unit Holding Cost (THB/unit/day) h_i
1	CHO	Chocolate	18	21	25	0.002044	0.1
2	CHP	Chili Paste	5	25	10	0.003288	0.1
3	DES	Dessert	5	15	135	0.002327	0.3
4	DRY	Dry Goods	15	25	44	0.023501	0.1
5	EGG	Egg	7	20	103	0.046069	0.1
6	FRO	Frozen Food	-20	0	169	0.032325	0.5
7	FRT	Fruit	5	13	254	0.044995	0.3
8	JIC	Juice	7	20	91	0.005463	0.1
9	MLK	Milk	10	20	1477	0.007719	0.1
10	NOD	Noodle	5	20	64	0.011067	0.1
11	PCM	Processed Meat	5	15	889	0.007894	0.3
12	PST	Pastry	20	25	878	0.005782	0.1
13	RDM	Ready Meal	15	20	185	0.003072	0.1
14	RWM	Raw Meat	-2	4	2397	0.039030	0.5
15	SAU	Sauce	15	25	37	0.005621	0.1
16	TFU	Tofu	15	25	407	0.012181	0.1
17	VEG	Vegetable	2	5	1573	0.026141	0.3
18	YOG	Yogurt	1	5	1342	0.005849	0.3

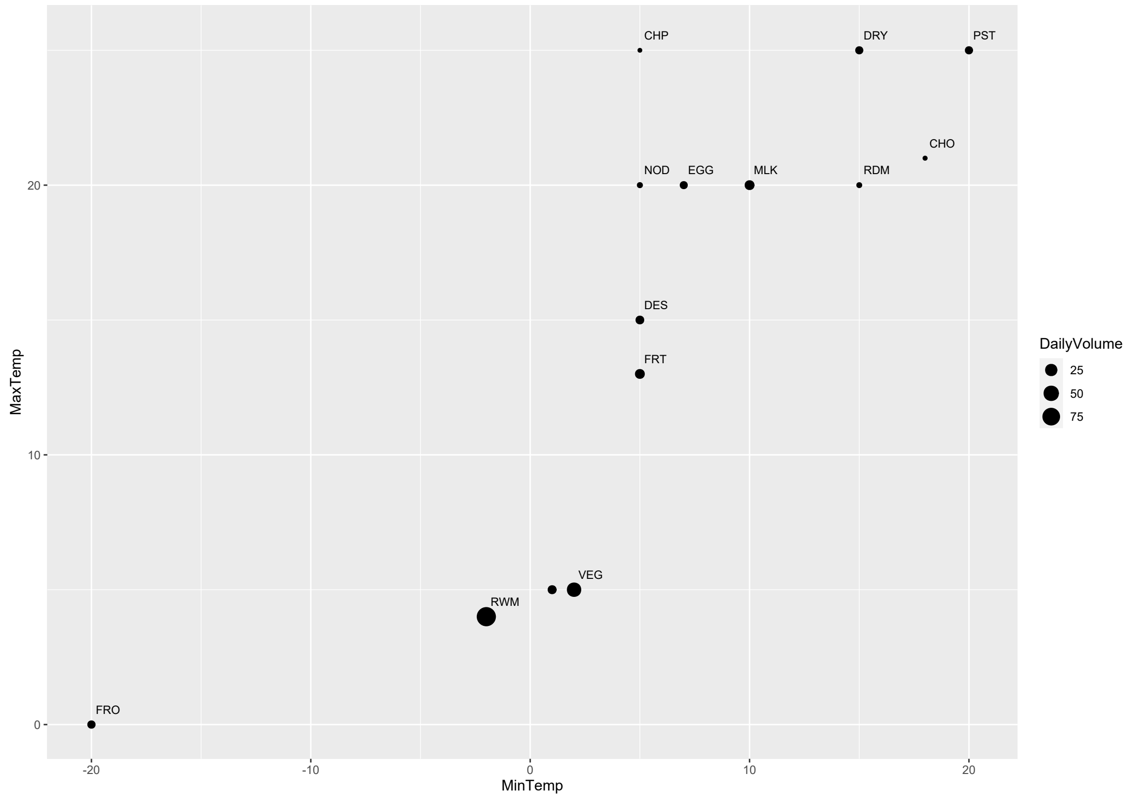

Fig. 2. Minimum and maximum temperatures as well as daily volumes of all items. The unit of temperature is °C, and that of the daily volume is cubic meter per day.

Table 4. Truck information.

Truck Type	Width x Length x Height ($m \times m \times m$)	Truck No.	Volume (m^3) κ_j	FTL Cost (THB/truck) K_j	(Min,Max) Temp. (°C) (y_j^L, y_j^U)
18W	2.50 x 12.00 x 2.00	$j \in \{1,2,3\}$	60	1022	(-5,15)
4W	1.70 x 3.12 x 1.70	$j \in \{4,5,6\}$	9	340	(-5,15)

$T_i = 1$ for all $i = 1, 2, \ldots, n$. Thus, their order size is equal to their daily demand: $q_i = d_i T_i$. Currently, one truck has one single temperature zone. In the case study, we will quantify the monetary benefit from using the state-of-the-art "multi-temp" truck. In the proposed policy, two bulkheads are used, and the multi-temp truck has three different temperature zones; see Fig. 1. We determine the temperature and the volume of each zones as well as the cycle time of each item so that the average daily total cost is minimized. In our numerical example, we write all models using AMPL and solve them using the BARON solver on the NEOS Server.

We consider two current policies, called C1 and C2, where the cycle times of all items are equal to one day. C1 uses 5 trucks (three 18W and two 4W trucks), whereas C2 uses 6 trucks (three 18W and three 4W trucks). We propose three policies, called P1, P2 and P3. For P1, the cycle times of all items are equal to one day, but each truck can have up to three temperature zones. For P2 and P3, we optimize the cycle time of each item. Each truck has one temperature zone for P2, whereas each truck can have up to three temperature zones for P3.

Deterministic demand

Assume that the demand is deterministic. The average daily total cost and key performance measures of all policies are provided in Table 5. The truck volume utilization for C1 with 5 trucks is slightly higher than that for C2 with 6 trucks. Since C1 uses one truck less than C2, the average daily FTL (major setup) cost for C1 is smaller than for C2. However, the average daily temperature penalty cost for C1 is higher than that for C2, since C2 has one more temperature zone (one truck), compared to C1. In our proposed policy P1, the cycle times of all items are equal to one day, but we allow each truck to have at most three temperature zones. P1 uses five trucks as in C1; thus, the FTL cost of P1 is identical to that of P2. In both P1 and P3, multi-temp trucks are used, their temperature penalties are the smallest. The temperature in each zone is shown in the bottom part of Table 5. For instance, for P1, the 1st 18-wheel truck has two zones with temperatures of 15 and 5 °C, and the 2nd 18-wheel truck has two zones with temperatures of 15 and 4 °C, and so on. P1 improve the current policies (C1 and C2) by using the multi-temp trucks while keeping the cycle time of one day for all items. Alternatively, we can improve the current policies (C1 and C2) by jointly optimizing the cycle times of all items while a single-temp truck is used: In P2, the optimal base cycle is $T = 0.81$ day, and the low-demand items (namely CHO, CHP, RDM and SAU) have longer cycle times. In the proposed policy P3, we optimize both the cycle time and the number of temperature zones; thus, P3 has the lowest average daily cost. For each policy, the cycle time of each item is provided in Table 6. For policies C1, C2 and P1, all items are replenished at the same time ($m_i = 1$ for all i), so its order size is equal to the daily demand ($q_i = d_i$ for all i). In P3, the optimal base cycle is 0.59 day or approximately 14 hours: For each item i, its cycle time is $T_i = m_i T$. For instance, consider Item $i = 3$ that has the lowest daily demand rate ($d_i = 10$ unit/day). It is replenished once every three base cycles, and its optimal cycle time is $T_i = 3(0.59) = 1.77$ day, which is the longest. Its order size has to cover the total demand during 1.77 day, i.e., $q_i = d_i T_i \approx 18$ unit.

Table 7 shows the allocation plan, i.e., the volume (given in cubic meter) and the temperature (given in °C) in each zone (denoted after the . symbol) in each truck for all five policies, namely C1, C2, P1, P2 and P3. For example, in P1 #4.3 denotes truck $j = 4$ (the first 4W truck) and zone 3, and #5.1 denotes truck $j = 5$ (the second 4W truck) and zone 1. For C1, C2 and P2, each truck has a single temperature zone, so there is no . symbol. The top three rows show the minimum and maximum temperature of all items. These rows show the input parameters, whereas the rest of the table show the decision variables. The decision variable y_{jk}, the temperature point of truck j in zone k, is provided in column Temp. The decision variable x_{ijk}, the allocation quantity of item i on truck j in zone k, is provided in column Volume Allocation. We put * next to the volume to indicate the temperature violation. For instance, P3 uses two 18W trucks: the 1st truck has two zones with temperatures 5 and 15 °C, and the 2nd truck has two zones with temperatures 15 and 0 °C. In the 1st truck and in the 2nd zone (denoted as#1.2), we allocate $x_{112} = 30$ cubic meters for Item 1, $x_{412} = 26$ cubic meters for Item 4, and so on. Item 1 incurs the temperature penalty, since its minimum temperature is 18 °C, but the temperature inside this zone is 15 °C.

Stochastic demand

For the stochastic demand, we consider the periodic-review OUTL system. Assume that the daily demand for item i is normally distributed with the mean of d_i and the standard deviation of $0.025d_i$; i.e., the standard deviation is 2.5% of the mean demand. Assume that backorder cost $b_i = 0$; in other words, an optimistic view is taken.

Suppose that the cycle time is fixed to one day for all items. We vary the cycle service level from 90% to 99.9% under different scenarios as shown in Fig. 3.

The expected daily total cost is smallest, when the lead time is negligible ($L = 0$), and the multi-temp

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Table 5. Summary of all policies for deterministic demand.

Policy	Cycle time = 1 day Single-temp C1	C2	Multi-temp P1	Optimal cycle time Single-temp P2	Multi-temp P3
Cycle time	1	1	1	Varies by items	
Max temp zones per truck	1	1	3	1	3
18-wheel trucks used	3	3	3	3	2
4-wheel trucks used	2	3	2	0	0
Truck volume utilization	0.990	0.947	0.990	0.889	1.000
Total temp zones	5	6	11	3	4
Total cost	9476	8188	7815	8003	7210
Temp penalty cost	3894	2266	2233	2571	2233
FTL cost	3946	4286	3946	4028	3800
Minor setup cost	200	200	200	223	291
Holding cost	1437	1437	1437	1181	887
Base cycle	1	1	1	0.811	0.591
Temp in 18W truck ($j = 1$)	15	-2	(15,5)	15	(15,5)
Temp in 18W truck ($j = 2$)	2	5	(15,4)	4	(15,0)
Temp in 18W truck ($j = 3$)	0	2	(7,-2)	1	N/A
Temp in 4W truck ($j = 4$)	1.00	15	(15,13,-2)	N/A	N/A
Temp in 4W truck ($j = 5$)	-2	10	(10,5)	N/A	N/A
Temp in 4W truck ($j = 6$)	N/A	15	N/A	N/A	N/A

Table 6. Cycle time in all policies.

Input parameter			C1,C2,P1 ($T = 1$)			P2 ($T = 0.81$)			P3 ($T = 0.59$)		
Item i	Code	d_i	q_i	m_i	T_i	q_i	m_i	T_i	q_i	m_i	T_i
1	CHO	25	25	1	1	81	4	3.24	30	2	1.18
2	CHP	10	10	1	1	57	7	5.68	18	3	1.77
3	DES	135	135	1	1	110	1	0.81	80	1	0.59
4	DRY	44	44	1	1	36	1	0.81	26	1	0.59
5	EGG	103	103	1	1	84	1	0.81	61	1	0.59
6	FRO	169	169	1	1	137	1	0.81	100	1	0.59
7	FRT	254	254	1	1	206	1	0.81	150	1	0.59
8	JIC	91	91	1	1	74	1	0.81	108	2	1.18
9	MLK	1477	1477	1	1	1198	1	0.81	873	1	0.59
10	NOD	64	64	1	1	52	1	0.81	76	2	1.18
11	PCM	889	889	1	1	721	1	0.81	525	1	0.59
12	PST	878	878	1	1	712	1	0.81	1037	2	1.18
13	RDM	185	185	1	1	300	2	1.62	219	2	1.18
14	RWM	2397	2397	1	1	1944	1	0.81	1416	1	0.59
15	SAU	37	37	1	1	90	3	2.43	44	2	1.18
16	TFU	407	407	1	1	330	1	0.81	241	1	0.59
17	VEG	1573	1573	1	1	1276	1	0.81	929	1	0.59
18	YOG	1342	1342	1	1	1089	1	0.81	793	1	0.59

Table 7. Allocation plan.

Item			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Min Temp			18	5	5	15	7	-20	5	7	10	5	5	20	15	-2	15	15	2	1
Max Temp			21	25	15	25	20	0	13	20	20	20	15	25	20	4	25	25	5	5
Policy	Truck.Zone $\#j.k$	Temp y_{jk}	Volume Allocation x_{ijk}																	
C1	18W #1	15	25*	10	135	44	103	-	254*	91	1477	64	889	878*	185	256*	37	407	-	-
	18W #2	2	-	-	-	-	-	-	-	-	-	-	-	-	-	458	-	-	1573	172
	18W #3	0	-	-	-	-	-	88	-	-	-	-	-	-	-	1465	-	-	-	-
	4W #4	1	-	-	-	-	-	-	-	-	-	-	-	-	-	56	-	-	-	1171
	4W #5	-2	-	-	-	-	-	82	-	-	-	-	-	-	-	163	-	-	-	-
C2	18W #1	-2	-	-	-	-	-	169	-	-	-	-	-	-	-	1117	-	-	-	-
	18W #2	5	-	10	135	-	34*	-	254	-	-	64	889	-	-	-	-	-	1191	1342
	18W #3	2	-	-	-	-	-	-	-	-	-	-	-	-	-	1281	-	-	383	-
	4W #4	15	25*	-	-	45	70	-	-	91	311	-	-	-	185	-	37	407	-	-
	4W #5	10	-	-	-	-	-	-	-	-	1166	-	-	878*	-	-	-	-	-	-
P1	18W #1.1	15	25*	-	135	-	-	-	-	-	1099	64	889	238*	185	-	37	292	-	-
	18W #1.2	5	-	-	-	-	-	-	23	-	-	-	-	-	-	-	-	-	1105	1342
	18W #2.1	15	-	-	-	23	-	-	-	-	-	-	-	-	-	-	-	116	-	-
	18W #2.2	4	-	-	-	-	-	-	-	-	-	-	-	-	-	1183	-	-	390	-
	18W #3.1	7	-	10	-	-	104	-	105	-	-	-	-	-	-	-	-	-	-	-
	18W #3.2	-2	-	-	-	-	-	169	-	-	-	-	-	-	-	1155	-	-	-	-
	4W #4.1	-2	-	-	-	-	-	-	-	-	-	-	-	-	-	60	-	-	-	-
	4W #4.2	13	-	-	-	-	-	-	44	91	-	-	-	-	-	-	-	-	-	-
	4W #4.3	15	-	-	-	22	-	-	-	-	-	-	-	641*	-	-	-	-	-	-
	4W #5.1	5	-	-	-	-	-	-	83	-	-	-	-	-	-	-	-	-	79	-
	4W #5.2	10	-	-	-	-	-	-	-	-	379	-	-	-	-	-	-	-	-	-
P2	18W #1	15	81*	57	110	36	84	-	206*	74	1198	52	721	712*	300	-	90	330	-	-
	18W #2	4	-	-	-	-	-	-	-	-	-	-	-	-	-	683	-	-	1276	-
	18W #3	1	-	-	-	-	-	137*	-	-	-	-	-	-	-	1261	-	-	-	1089
P3	18W #1.1	5	-	18	80	-	-	-	150	-	-	76	525	-	-	-	-	-	929	793
	18W #1.2	15	30*	-	-	26	61	-	-	108	675	-	-	1037*	219	-	44	241	-	-
	18W #2.1	15	-	-	-	-	-	-	-	-	198	-	-	-	-	-	-	-	-	-
	18W #2.2	0	-	-	-	-	-	100	-	-	-	-	-	-	-	1416	-	-	-	-

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Table 8. Summary of all policies for stochastic demand.

Policy	Cycle time = 1 day		Optimal cycle time	
	Single-temp C2′	Multi-temp P1′	Single-temp P2′	Multi-temp P3′
Cycle time	1	1	Varies by items	
Max temp zones per truck	1	3	1	3
18-wheel trucks used	3	3	3	3
4-wheel trucks used	3	3	0	0
Truck volume utilization	1.000	0.950	0.930	1.000
Total temp zones	6	15	3	7
Total cost	8469	8405	8358	7817
Temp penalty cost	2406	2342	2625	2438
FTL cost	4286	4286	4253	3787
Minor setup cost	200	200	218	200
Holding cost	1577	1577	1262	1392
Base cycle	1	1	0.768	0.862
Temp in 18W truck ($j = 1$)	5	(15,5)	0	(1,4,15)
Temp in 18W truck ($j = 2$)	4	4	15	(5,15)
Temp in 18W truck ($j = 3$)	-2	(4,-5)	3	(4,15)
Temp in 4W truck ($j = 4$)	15	(15,-5)	N/A	N/A
Temp in 4W truck ($j = 5$)	15	(15,-5)	N/A	N/A
Temp in 4W truck ($j = 6$)	15	15	N/A	N/A
Δ Total cost	281	590	355	607

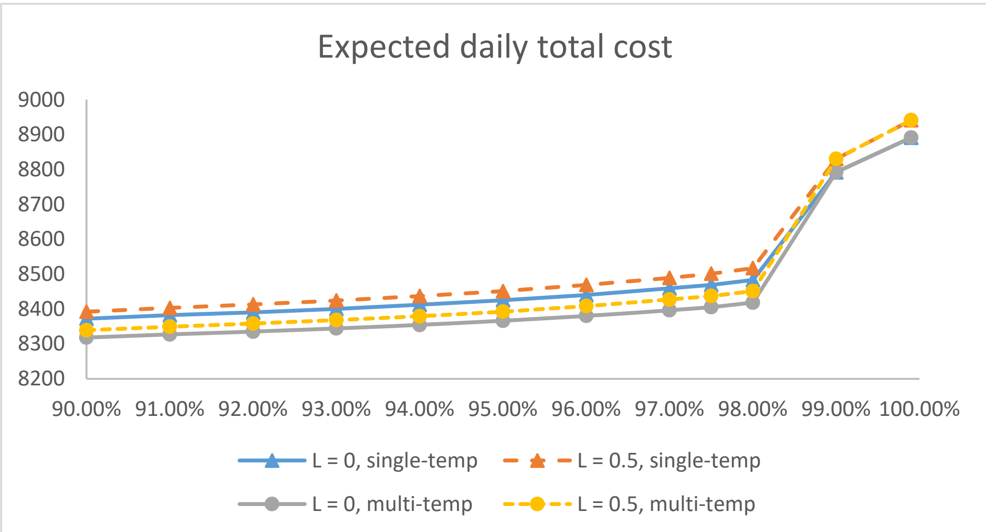

Fig. 3. Expected daily total costs under different scenarios.

trucks are used. It is largest, when the lead time is positive (here, $L = 0.5$ day), and the single-temp trucks are used. Reducing the lead time and using the multi-temp truck can reduce the cost. In this case study, as we can see from Fig. 3, the cost saving from multi-temp truck (the yellow line) is larger than that of lead time reduction (the blue line). Furthermore, Fig. 3 clearly shows the trade-off between cost and service level. As the cycle service level increases, we need to keep more safety stock, and the expected cost increases.

For the rest of this section, we assume that lead time is zero and that the safety factor is 1.96, which corresponds to the cycle service level of 97.5%. In analogous to C2, P1, P2 and P3 in the case of deterministic demand, we consider the current policy C2′ and propose three policies P1′, P2′ and P3′. As in C2, C2′ fixes the cycle time to be one day for all items and uses single-temp trucks. As in P1, P1′ fixes the cycle time to be one day for all items but uses multi-temp trucks. As in P2, P2′ optimizes the cycle time and uses single-temp trucks. As in P3, P3′ optimizes both the cycle time and uses multi-temp trucks. Recall that the truck utilization of C1 is larger than that of C2, but the average total cost associated with C1 is smaller than that with C2; thus, for the case of stochastic demand, we do not consider C1, and we will see that the truck utilization for C2′ is 100 percent.

We summarize the performance measures and temperature points of all four policies in Table 8. For C2′, the temperatures inside the 18W trucks are 5, 4 and -2 °C, and the temperature inside the 4W truck is 15 °C. For P1′, the first 18W truck has two temperature zones, where temperatures are 15 and 5 °C. The second 18W truck has one temperature zone with 4 °C. The third 18W truck has two temperature zones, where temperatures are 4 and -5 °C. The first two 4W trucks ($j = 4, 5$) have two temperature zones, where temperatures are 15 and -5 °C, and the last 4W truck ($j = 6$) has one temperature zone with 15 °C. In C2′ and P1′, the cycle time is fixed to be 1 day, whereas in P2′ and P3′, the cycle time is optimized. The optimal base cases in P2′ and P3′ are 0.768 and 0.862 day, respectively. In P2′ and P3′, we use only 18W trucks ($j = 1, 2, 3$). For P2′, the temperatures inside the 18W trucks are 0, 15 and 3 °C. For P3′, the first 18W truck has three temperature zones, where temperatures are 1, 4 and 15 °C. The second 18W truck has two temperature zones, where temperatures are 5 and 15 °C. The last 18W truck has two temperature zones, where temperatures are 4 and 15 °C. The last row in Table 8 shows the increase in total cost due to demand uncertainty. Suppose that the cycle time is one day and that the single-temp trucks are used. Then, the increase in total cost due to demand uncertainty is the difference between the total cost for stochastic demand (Column C2′ in Table 8) and that for deterministic demand (Column C2 in Table 5), $8469 - 8188 = 281$. This is the smallest increase among all four cases. The largest increase of 607 occurs when the cycle time is optimized and the multi-temp trucks are used; the difference between the total cost for stochastic demand (Column P3′ in Table 8) and that for deterministic demand (Column P3 in Table 5) is $7817 - 7210 = 607$.

5. Conclusion

In summary, we propose a CJRP with refrigerated FTL delivery. The truck capacity is formulated as a hard constraint, whereas the temperature target is formulated as a soft constraint. In the deterministic model, we determine the cycle time of each product, the temperature of each zone in each truck, and the allocation plan that specifies how many units of each product would be delivered in each zone in each truck. For each product with constant demand, the order quantity remains the same for all cycles. When demand is random, the order quantity of each product is determined from the order-up-to level and its current inventory position. In our stochastic model, the order-up-to level is chosen to maintain the desired in-stock probability of each product. In the case study, our model is applied to solve the joint replenishment problem at one of the largest modern grocery retailers in Thailand. We quantify the monetary benefit from using the multi-temp truck and the lead time reduction initiative.

A few extensions are as follows. In the current model, we assume that all units of each product are identical. We could extend our model to capture aging products: The old aging units may be more sensitive to the temperature violation than the new units. Furthermore, the customer demand, especially for perishable products, usually depends on the product quality, which can be affected by both the age and the temperature violation. In the constrained JRP, items could be picked up from different locations, we could include the routing decision, and the temperature would be dynamic, since the truck temperature at the beginning of the journey (when only few items have been picked up by the truck) could be different from that at the end of the journey (when all items have been picked up). We hope to pursue these or related issues in the future.

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