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Distributed Model Reference Adaptive Control for Vehicle Platoons with Uncertain Dynamics

Agung Prayitno[a] and Itthisek Nilkhamhang[b,*]

School of Information, Computer, and Communication Technology, Sirindhorn International Institute of Technology, Thammasat University, 131 Moo 5, Tiwanon Road, Bangkadi, Muang, Pathum Thani, 12000, Thailand
E-mail: [a]d6222300029@g.siit.tu.ac.th, [b,*]itthisek@siit.tu.ac.th (Corresponding author)

Abstract. This paper proposes a distributed model reference adaptive controller (DMRAC) for vehicle platoons with constant spacing policy, subjected to uncertainty in control effectiveness and inertial time lag. It formulates the uncertain vehicle dynamics as a matched uncertainty, and is applicable for both directed and undirected topologies. The directed topology must contain at least one spanning tree with the leader as a root node, while the undirected topology must be static and connected with at least one follower receiving information from the leader. The proposed control structure consists of a reference model and a main control system. The reference model is a closed-loop system constructed from the nominal model of each follower vehicle and a reference control signal. The main control system consists of a nominal control signal based on cooperative state feedback and an adaptive term. The nominal control signal allows the followers cooperatively track the leader, while the adaptive term suppresses the effects of uncertainties. Stability analysis shows that global tracking errors with respect to the reference model and with respect to the leader are asymptotically stable. The states of all followers synchronize to both the reference and leader states. Moreover, with the existence of unknown external disturbances, the global tracking errors remain uniformly ultimately bounded. The performance of the controlled system is verified through the simulations and validates the efficacy of the proposed controller.

Keywords: Distributed model reference adaptive control, cooperative tracking control, multi-agent systems, uncertain systems, vehicle platoon.

1. Introduction

Rapid advancements in self-driving cars, smart sensors, and communication technology have led to the development of cooperative vehicle control [1]. An example with high application potential is the vehicle platoon, in which multiple networked vehicles move with synchronized velocity and acceleration, while maintaining separation distance. The formation typically has a lead vehicle that generates a reference trajectory for all followers. This is suitable for applications where a number of vehicles depart together towards a common location. Examples include container trucks carrying products from warehouse to port or travel buses that transport tourists to their destinations. The development of vehicle platoons for passenger vehicles may also in beneficial for intelligent highway systems.

The current state of research on vehicle platoons is summarized in survey papers [1, 2], including problems and challenges in development and implementation. In general, a vehicle platoon consists of four main components: longitudinal vehicle dynamics; formation geometry; information flow topology; and a distributed controller [3]. The longitudinal vehicle dynamics can be represented using first-order [1], second-order [4], third-order [5], or nonlinear models [6]. In addition, the vehicles in the platoon may be considered as homogeneous or heterogeneous [7]. Formation geometry generally refers to the spacing policy employed by the vehicle platoon, with the most common being constant spacing policy (CSP) [5] and constant time heading (CTH) [4]. Information flow topology describes how communication occurs between vehicles. Conventional topologies include predecessor-follower (PF) [5, 8], predecessor-follower leader (PFL) [4, 9], bi-directional [BD] [10], bi-directional leader (BDL) [11], two-predecessor following (TPF) [5] and two-predecessor following leader (TPFL) [5]. Lastly, each vehicle must implement a distributed controller that maintains the desired intervehicle distance and synchronization to the leader, while addressing problems encountered in real applications, such as constraints on state and control input [12], actuator saturation [13], communication issues such as delay [4] and loss [8], switching topology [14], uncertain vehicle dynamics [8, 15], and external disturbances [16]. Examples of distributed controllers that have been designed for vehicle platoons include cooperative state variable feedback control (SVFB) [5], adaptive control [6, 8] and robust control [17].

A vehicle platoon can be considered as a multi-agent system. The inclusion of a lead vehicle implies that it is categorized as a type of leader-follower consensus [18]. In certain literatures, it is also referred to as cooperative tracking [19]. The information exchange between vehicles can be modeled using communication graph theory as directed (PF, PFL, TPF, TPFL) [5] or undirected topologies (BD, BDL) [20]. In a directed topology, a vehicle can send information to its neighbor(s) but not necessarily vice versa [21], while in an undirected topology, the communication between a vehicle and its neighbor(s) is bidirectional with equal weight [19].

Information exchange between vehicles allows the implementation of distributed controllers in order to achieve leader-follower consensus. Most strategies rely on a nominal model of the vehicle dynamics. Zhang et.al [22] proposed LQR-based cooperative state variable feedback (SVFB), cooperative observer design and cooperative output feedback (OPFB) for leader-follower consensus when the agents have identical dynamics. Meanwhile, Li et.al [23] designed a fully distributed adaptive consensus protocol in which the coupling weight is adapted automatically without information regarding the communication graph. Moreover, Li et.al [24] introduced distributed consensus using node-based and edge-based adaptive protocols for both leaderless and leader-follower consensus. Zegers et.al [10] proposed a distributed consensus control for vehicle platoons with velocity-dependent spacing policy and bidirectional topology.

However, an automotive vehicle is a complex system composed of the engine, driveline, brake system, aerodynamic drag, tire friction, rolling resistance, and gravitational force [7]. Therefore, it can be difficult, or even impossible, to obtain a perfect model that describes all the dynamics. Instead, most modelling processes rely on simplifications, approximations, and assumptions [25]. The resulting nominal models often include modeling errors or uncertainties, and may not accurately reflect the real dynamics of the vehicle [26]. Uncertainties in the model may be considered as structured or unstructured, and are caused by operating parameters, such as vehicle age, chassis, road conditions, and weather [7, 27, 28]. A conventional controller designed using the nominal model of a vehicle will experience a decline in performance as a result of uncertain dynamics, and even lead to instability of the closed loop system [7, 26]. In addition, the effects of uncertainty in a vehicle platoon may be propagated upstream or downstream, depending on the topology used [29].

Several distributed controllers for vehicle platoons with uncertainty have been developed. Zhou et.al [27] designed a controller that consists of an upper-level control for maintaining spacing distance and speed, and a lower-level control to deal with uncertain dynamics, where the ratio of desired acceleration and inertial time lag is treated as a bounded, time-varying parameter. Zou et.al [28] proposed a self-tuning algorithm for velocity and position control of a vehicle platoon with parameter uncertainties. Harfouch et.al [8] have proposed a model reference adaptive control for heterogeneous vehicle platoons, where the heterogeneity is treated as a homogeneous vehicle platoon with uncertainty in engine performance (control effectiveness) and driveline time constant (inertial time lag). The dynamics of the leader is used as the known nominal model. However, the controllers in [8, 27, 28] are designed for a specific directed topology (PF) and cannot be applied to other topologies.

The issue of uncertainties for general leader-follower consensus has been addressed by some literatures. Peng et.al [15] proposed a distributed model reference adaptive control to deal with unknown matched uncertainty and matched disturbance. However, this controller is designed specifically for undirected topologies and utilizes the leader state as a reference, which may not be practical for vehicle platoon application. Song et.al [30] proposed a distributed adaptive controller for leader-follower consensus that consider parametric uncertainties in both the leader and followers, based on the assumption that the leader input is locally known. Peng et al [31] achieved distributed adaptive synchronization based on neural network for directed and undirected topologies with unknown matched uncertainties. However, they required knowledge about the bound of the basis functions to design the control parameters and did not consider uncertainties in the control effectiveness.

This paper proposes a distributed model reference adaptive controller (DMRAC) for vehicle platoons with constant spacing policy, subjected to uncertainty in control effectiveness and inertial time lag. The uncertain vehicle dynamics is formulated as a matched uncertainty, which is a type of structured uncertainty that can be matched by the control signal [32]. The proposed controller is applicable for both directed and undirected topologies, and consists of a reference model and a main control system. The reference model is a closed-loop system, constructed using the nominal model and a reference control signal, that generates a reference state. The main control system involves two terms, namely (i) a nominal control signal based on cooperative state feedback and (ii) an adaptive term. The nominal control signal is responsible for synchronizing the followers to the leader, while the adaptive term suppresses the effects of uncertainties in each follower. The main contributions of this paper can be summarized as follows:

(i) The proposed DMRAC does not require knowledge about the dynamics of the lead vehicle to be available to all followers, as compared to [15, 31]. Additionally, the controller is robust to uncertainties in control effectiveness.

(ii) The adaptive controller can be applied to a vehicle platoon with both directed and undirected topologies, as long as the given condition on the scalar coupling gain is satisfied. The directed topology must contain at least one spanning tree with the leader as a root node, while the undirected topology must be static and connected with at least one follower receiving information from the leader. This removes the topology limitations found in [8, 27, 28].

The rest of the paper presents the details of the proposed controller, including stability and consensus analysis. The performance of a vehicle platoon with DMRAC is validated through numerical simulations.

2. Information Flow Graph and Problem Formulation

2.1. Information Flow Graph

The information exchange between vehicles is represented by a graph. The graph is denoted as $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is a set of nodes that represents the vehicles and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges representing the information exchange between vehicles. In a leader-follower system, the exchange of information between followers can be represented by an adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$. Each entry of the adjacency matrix has the value: $a_{ij} = 1$ if and only if $\{v_j, v_i\} \in \mathcal{E}$, otherwise $a_{ij} = 0$. Here, $\{v_j, v_i\} \in \mathcal{E}$ signifies that vehicle i can receive information from vehicle j. The number of neighbors that can send information to vehicle i is determined by the in-degree matrix, $D = diag\{d_{11}, d_{22}, \ldots, d_{NN}\}$, where $d_{ii} = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix L is related to graph $\mathcal{G}$ and is defined as $L = D - \mathcal{A} \in \mathbb{R}^{N \times N}$ with diagonal elements $\ell_{ii} = d_{ii}$ and the other elements given by $\ell_{ij} = -a_{ij}$. A pinning gain matrix represents the information flow from the leader to the followers and is expressed as $G = diag\{g_{11}, g_{22}, \ldots, g_{NN}\}$, where $g_{ii} = 1$ means that follower i can receive information directly from the leader, otherwise $g_{ii} = 0$. A graph is undirected if all edges are bidirectional and with equal weights for all i and j, i.e., $a_{ij} = a_{ji}$. Meanwhile, a directed graph (digraph) is a graph where all edges are directed from one node to another. A digraph contains a spanning tree if there is a root node, and departing from this root node, all nodes can be reached by following edge arrows. An augmented graph $\tilde{\mathcal{G}}(\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ is defined such that $\tilde{\mathcal{V}} = \{v_0, v_1, v_2, \ldots, v_N\}$ and $\tilde{\mathcal{E}} \subseteq \tilde{\mathcal{V}} \times \tilde{\mathcal{V}}$.

Lemma 1: *If the augmented graph $\tilde{\mathcal{G}}$ is directed and contains at least one spanning tree with the leader as a root node, then $(L + G)$ is a nonsingular M-matrix and we can define*

$$F = [f_1, f_2, \cdots, f_N]^T = (L + G)^{-1}\mathbf{1},$$
$$S = diag\left\{\frac{1}{f_1}, \frac{1}{f_2}, \cdots, \frac{1}{f_N}\right\},$$
$$T = S(L + G) + (L + G)^T S, \tag{1}$$

where $\mathbf{1} = col(1, 1, \cdots, 1) \in \mathbb{R}^N$, then $S > 0$ and $T > 0$. That is, there exists a positive diagonal matrix S such that $T = S(L + G) + (L + G)^T S$ is positive definite [31], [33], [34].

Lemma 2: *If the graph $\mathcal{G}$ is undirected, static, and connected, where at least one follower receives information from the leader, then $(L + G)$ is positive definite [15].*

2.2. Problem Formulation

The longitudinal dynamics of each agent in a vehicle platoon, shown in Fig. 1, can be represented as a linearized third-order system [5, 27],

$$\begin{aligned}
\dot{p}_i(t) &= v_i(t), \\
\dot{v}_i(t) &= a_i(t), \\
\dot{a}_i(t) &= -\frac{1}{\tau_i}a_i(t) + \frac{\Omega_i}{\tau_i}u_i(t),
\end{aligned} \tag{2}$$

where $p_i(t)$, $v_i(t)$, and $a_i(t)$ are the position, velocity, and acceleration of vehicle i respectively. $u_i(t), \tau_i, \Omega_i$ are the control input, inertial time lag of the powertrain and control effectiveness of vehicle i. The nominal value for the inertial time lag of the powertrain is τ and the nominal value for control effectiveness is 1. The lead vehicle is denoted by $i = 0$ and followers are denoted by $i = 1,2,\dots,N$. For readability, the time notation (t) is omitted in the remaining derivations.
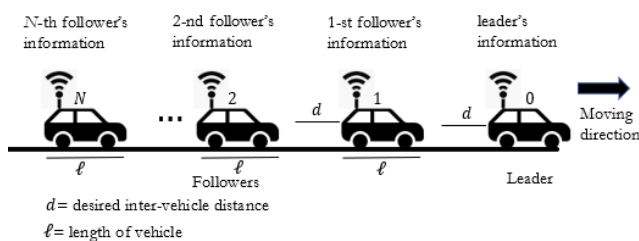

Fig. 1. A vehicle platoon with 1 leader and N followers.

The followers are subjected to uncertain dynamics and can be represented in the state-space form as

$$\dot{x}_i = Ax_i + B\Omega_i\big[u_i + \Omega_i^{-1}\eta_i(x_i)\big], \tag{3}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, $\Omega_i, \eta_i(x_i)$ are the state vector, control input, control effectiveness and unknown matched uncertainty of vehicle i respectively. $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are the nominal vehicle system matrices given as

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix}, \tag{4}$$

Assumption 1: *The unknown matched uncertainty in (3) is parameterized as [25]*

$$\eta_i(x_i) = W_i^T\sigma_i(x_i) \tag{5}$$

where $W_i \in \mathbb{R}^{s \times m}$ is an unknown constant weighting matrix and $\sigma_i(x_i): \mathbb{R}^n \to \mathbb{R}^s$ is a known basis vector function.

Assumption 2: *The leader is assumed to have nominal control effectiveness $(\Omega_0 = 1)$, no uncertainty in the inertial time lag of the powertrain $(\tau_0 = \tau)$ and moves at a constant speed $(u_0 = 0)$.*

Based on Assumption 2, the dynamics of the leader can be represented in the state-space form as

$$\dot{x}_0 = Ax_0, \tag{6}$$

where $x_0 \in \mathbb{R}^n$ is the leader's state.

In this paper, the state vector is defined as $x_i = [p_i + i \cdot d_i \quad v_i \quad a_i]^T$, where $d_i = d$ is the desired constant spacing distance.

Remark 1: The control effectiveness Ω_i may deviate from the nominal value as the vehicle moves uphill or downhill, encounters sudden wind gusts or experiences mechanical wear [8].

Remark 2: The vehicle dynamics (3) results in a semi-homogeneous vehicle platoon that becomes homogenous if there are no uncertainties ($\Omega_i = 1$ and $\eta_i(x_i) = 0$) [25]. This can be used to describe a heterogeneous vehicle platoon, which would be represented as a homogeneous vehicle platoon with uncertainty and considering the leader dynamics as the nominal model [8].

A reference model is constructed using the nominal model of each follower as

$$\dot{x}_{i,r} = Ax_{i,r} + Bu_{i,nr}, \tag{7}$$

where $x_{i,r} \in \mathbb{R}^n$ is the follower's reference state and $u_{i,nr} \in \mathbb{R}^m$ is the reference control signal.

The objective of this paper is to design a distributed controller u_i for each follower (3) such that the uncertain vehicle can track the reference model (7) and simultaneously achieve synchronization to the leader's state (6).

3. Distributed Model Reference Adaptive Control

The proposed controller is inspired by the success of LQR-based cooperative SVFB for multiagent systems with N homogeneous follower agents and a single leader, based on a nominal model [22]. In practice, a vehicle platoon may experience uncertainties that cause the vehicular dynamics of the followers to deviate significantly. A distributed controller designed using only the nominal model would therefore experience a deterioration in performance. In some cases, this can lead to instability of the entire system. The main purpose of this work is therefore to suppress the effects of uncertainties in a vehicle platoon.

The proposed distributed model reference adaptive controller (DMRAC) in Fig. 2 consists of a reference model and a main control system. The reference model is

a closed-loop system constructed using the nominal model and a reference control signal ($u_{i,nr}$) that generates a reference state ($x_{i,r}$). The main control system involves two terms, namely (i) a nominal control signal ($u_{i,n}$) based on cooperative state feedback and (ii) an adaptive term ($u_{i,a}$) that utilizes the tracking error w.r.t the reference model (e_i), the nominal control signal ($u_{i,n}$) and the vehicle state (x_i). The nominal control signal is responsible for synchronizing the followers to the leader, while the adaptive term suppresses the effects of uncertainties in each follower.
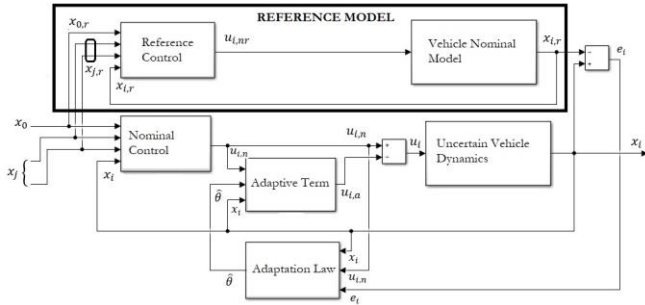

Fig. 2. Distributed model reference adaptive control for cooperative tracking.

Remark 3: The reference model uses the neighbors' actual state in place of the neighbors' reference state ($x_{j,r} = x_j$ and $x_{0,r} = x_0$).

3.1. Reference Model

The reference control signal of each follower vehicle is designed according to

$$u_{i,nr} = cK\varepsilon_{i,r}, \tag{8}$$

where c is a scalar coupling gain, $K \in \mathbb{R}^{m \times n}$ is the feedback gain matrix and $\varepsilon_{i,r} \in \mathbb{R}^n$ is the cooperative tracking error of the reference model defined as

$$\varepsilon_{i,r} = \sum_{j=1}^{N} a_{ij}(x_{j,r} - x_{i,r}) + g_{ii}(x_{0,r} - x_{i,r}). \tag{9}$$

The feedback gain matrix can be chosen as

$$K = R^{-1}B^T P. \tag{10}$$

Here, P is a solution of the algebraic Riccati equation (ARE)

$$0 = A^T P + PA + Q - PBR^{-1}B^T P, \tag{11}$$

where $Q = Q^T \in \mathbb{R}^{n \times n} > 0$ and matrix $R = R^T \in \mathbb{R}^{m \times m} > 0$.

By substituting the reference control (8) into (7), the closed-loop reference model of vehicle i becomes

$$\dot{x}_{i,r} = Ax_{i,r} + cBK\{\sum_{j=1}^{N} a_{ij}(x_{j,r} - x_{i,r}) + g_{ii}(x_{0,r} - x_{i,r})\} \tag{12}$$

3.2. Main Control System

The control input for vehicle i with the uncertain dynamics described by (3) is designed as

$$u_i = u_{i,n} - u_{i,a}, \tag{13}$$

where $u_{i,n}$ is the nominal control and $u_{i,a}$ is the adaptive term. The nominal control signal is designed using cooperative SVFB as

$$u_{i,n} = cK\varepsilon_i, \tag{14}$$

where $\varepsilon_i \in \mathbb{R}^n$ is the cooperative tracking error of the uncertain vehicle defined as

$$\varepsilon_i = \sum_{j=1}^{N} a_{ij}(x_j - x_i) + g_{ii}(x_0 - x_i). \tag{15}$$

The feedback gain matrix K is given by (10), and the condition on the coupling gain c will be discussed later.

Substituting (13) into (3), then adding and subtracting the term $cBK\varepsilon_i$ yields

$$\dot{x}_i = Ax_i + B\Omega_i[u_{i,n} - u_{i,a} + \Omega_i^{-1}\eta_i(x_i)] + cBK\varepsilon_i - cBK\varepsilon_i. \tag{16}$$

This can be re-expressed as

$$\dot{x}_i = Ax_i + cBK\varepsilon_i + B\Omega_i\left[u_{i,n} - u_{i,a} + \Omega_i^{-1}W_i^T\sigma_i(x_i) - \Omega_i^{-1}[u_{i,n}]\right]. \tag{17}$$

Finally, substituting (14) and (15) obtains

$$\dot{x}_i = Ax_i + cBK\{\sum_{j=1}^{N} a_{ij}(x_j - x_i) + g_{ii}(x_0 - x_i)\} + B\Omega_i[\theta_i^T\Phi_i(\sigma_i(x_i), u_{i,n}) - u_{i,a}], \tag{18}$$

where

$$\theta_i^T = \begin{bmatrix} \Omega_i^{-1}W_i^T \\ I - \Omega_i^{-1} \end{bmatrix}^T, \tag{19}$$

and

$$\Phi_i(\sigma_i(x_i), u_{i,n}) = \begin{bmatrix} \sigma_i(x_i) \\ u_{i,n} \end{bmatrix}. \tag{20}$$

From (12) and (18), it is seen that the state tracking error $(x_i - x_{i,r}) \to 0$ as $t \to \infty$ if $u_{i,a} = \theta_i^T\Phi_i(\sigma_i(x_i), u_{in})$. However, since θ_i^T is unknown, the estimated value $\hat{\theta}_i^T$ is used instead to construct the adaptive term as

$$u_{i,a} = \hat{\theta}_i^T\Phi_i(\sigma_i(x_i), u_{i,n}), \tag{21}$$

By substituting (21) into (18), the closed-loop uncertain vehicle dynamics becomes

$$\dot{x}_i = Ax_i + cBK\left\{\sum_{j=1}^N a_{ij}(x_j - x_i) + g_{ii}(x_0 - x_i)\right\} - B\Omega_i\left[\tilde{\theta}_i^T \Phi_i(\sigma_i(x_i), u_{i,n})\right], \quad (22)$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$ is the parameter estimation error and $\sigma_i(x_i) = x_i$ is the known basis function.

3.3. Global Dynamics Notation

For the purpose of stability analysis, the system is represented using global notations. The global dynamics of the leader is

$$\underline{\dot{x}}_0 = (I_N \otimes A)\underline{x}_0, \quad (23)$$

where $\underline{x}_0 = col(x_0, x_0, \cdots, x_0) \in \mathbb{R}^{nN}$ and $I_N \in \mathbb{R}^{N\times N}$ is the identity matrix and $\otimes$ is the Kronecker product.

The global closed-loop reference model is

$$\dot{x}_r = (I_N \otimes A - c(L+G) \otimes BK)x_r + (c(L+G) \otimes BK)\underline{x}_{0,r}, \quad (24)$$

where $x_r = col(x_{1,r}, x_{2,r}, \cdots, x_{N,r}) \in \mathbb{R}^{nN}$, $\underline{x}_{0,r} = col(x_{0,r}, x_{0,r}, \cdots, x_{0,r}) \in \mathbb{R}^{nN}$, L and G are the Laplacian matrix and the pinning gain matrix associated with the topology.

The global uncertain vehicle dynamics is

$$\dot{x} = (I_N \otimes A - c(L+G) \otimes BK)x + (c(L+G) \otimes BK)\underline{x}_0 - (I_N \otimes B)\Omega[\tilde{\theta}^T\Phi(x, u_n)]. \quad (25)$$

where $x = col(x_1, x_2, \cdots, x_N) \in \mathbb{R}^{nN}$, $\Omega = diag\{\Omega_1, \Omega_2, \cdots, \Omega_N\}$, $\tilde{\theta} = diag\{\tilde{\theta}_1, \tilde{\theta}_2, \cdots, \tilde{\theta}_N\}$ and $\Phi(x, u_n) = col(\Phi_1(x_1, u_{1,n}), \Phi_2(x_2, u_{2,n}), \cdots, \Phi_N(x_N, u_{N,n}))$.

Denote the global tracking error of all vehicles with respect to the reference model state as $e = x - x_r$ where $e = col(e_1, e_2, \cdots, e_N) \in \mathbb{R}^{nN}$. According to Remark 3, since $\underline{x}_{0,r} = \underline{x}_0$, then the global tracking error dynamics w.r.t the reference model is

$$\dot{e} = (I_N \otimes A - c(L+G) \otimes BK)e - (I_N \otimes B)\Omega[\tilde{\theta}^T\Phi(x, u_n)]. \quad (26)$$

Since the objective of the adaptive term is to suppress the effects of system uncertainties such that the state of each follower vehicle approaches the state of the reference model, $x \to x_r$, and knowing that $\underline{x}_0 = \underline{x}_{0,r}$, then for simplicity the global tracking error w.r.t the leader can be represented as $\delta = x_r - \underline{x}_0$, where $\delta = col(\delta_1, \delta_2, \cdots, \delta_N) \in \mathbb{R}^{nN}$. The global tracking error dynamics w.r.t to the leader is

$$\dot{\delta} = (I_N \otimes A - c(L+G) \otimes BK)\delta. \quad (27)$$

Let the global cooperative tracking error of the reference model be defined as $\varepsilon_r = -((L+G) \otimes I_n)\delta$, where $\varepsilon_r = col(\varepsilon_{1,r}, \varepsilon_{2,r}, \cdots, \varepsilon_{N,r}) \in \mathbb{R}^{nN}$. Thus, the global cooperative tracking error dynamics w.r.t the leader becomes

$$\dot{\varepsilon}_r = (I_N \otimes A - c(L+G) \otimes BK)\varepsilon_r. \quad (28)$$

4. Main Results and Stability Analysis

4.1. Vehicle Platoon with Directed Topology

Theorem 1. *Consider a vehicle platoon with the dynamics expressed by (3) and (6). The network topology is assumed to be directed and contains at least one spanning tree with the leader as a root node. The reference model is constructed according to (7) and (8). By applying the distributed controller (13) with feedback gain K as in (10) and selecting the coupling gain c such that*

$$c \geq \frac{1}{\min_{i=1\cdots N}(f_i\mu_i)}, \quad (29)$$

where μ_i is the ith eigenvalue of matrix T, and f_i is the ith row of column vector F defined in (1), along with the adaptation law

$$\dot{\hat{\theta}}_i = \gamma s_i \Phi_i(x_i, u_{in})e_i^T PB, \quad (30)$$

where $\gamma > 0$ is the adaptation rate, s_i is the ith eigenvalue of matrix S defined in (1), then the global tracking error w.r.t the reference state satisfies

$$\lim_{t\to\infty}\|e\| = 0, \quad (31)$$

and the global tracking error w.r.t the leader state satisfies

$$\lim_{t\to\infty}\|\delta\| = 0. \quad (32)$$

Proof. The stability proof of the system is conducted in two steps. Firstly, it will be shown that the uncertain vehicle can track the reference model, such that $e \to 0$ as $t \to \infty$. Secondly, the followers are guaranteed to synchronize to leader state, such that $\delta \to 0$ as $t \to \infty$.

4.1.1. Proof of $e \to 0$ as $t \to \infty$

Consider the following Lyapunov candidate function

$$V_1(e, \tilde{\theta}) = e^T(S \otimes P)e + \gamma^{-1}tr(\Omega^{1/2}\tilde{\theta}^T\tilde{\theta}\Omega^{1/2}), \quad (33)$$

The first derivative of V_1 along (26) is

$$\dot{V}_1 = e^T[S \otimes (PA + A^TP) - c\{S(L+G) + (L+$$

$$G)^T S\} \otimes PBR^{-1}B^T P]e - 2e^T\{(S \otimes PB)\Omega[\tilde{\theta}^T\Phi(x, u_n)]\} + 2\gamma^{-1}tr\left(\Omega\tilde{\theta}^T\dot{\hat{\theta}}\right). \quad (34)$$

Using Lemma 1,

$$\dot{V}_1 = e^T[S \otimes (PA + A^T P) - cT \otimes PBR^{-1}B^T P]e - 2\gamma^{-1}tr(\Omega\tilde{\theta}^T\left[\gamma\Phi(x, u_n)e^T(S \otimes PB) - \dot{\hat{\theta}}\right]), \quad (35)$$

where T is positive definite. There exists a unitary matrix J such that $J^T T J = diag\{\mu_1, \mu_2, \cdots, \mu_N\}$. Using this property,

$$\dot{V}_1 = \sum_{i=1}^{N} s_i e_i^T \{PA + A^T P - cf_i\mu_i PBR^{-1}B^T P\}e_i - \sum_{i=1}^{N} 2\gamma^{-1}tr(\gamma s_i\Omega_i\tilde{\theta}_i^T\left[\Phi_i(x_i, u_{i,n})e_i^T PB\right] - \dot{\hat{\theta}}_i). \quad (36)$$

By choosing a coupling gain c that satisfies (29) and the adaptation law $\dot{\hat{\theta}}_i$ according to (30),

$$\dot{V}_1 \leq \sum_{i=1}^{N} s_i e_i^T \{PA + A^T P - PBR^{-1}B^T P\}e_i, \quad (37)$$

or alternatively

$$\dot{V}_1 \leq - \min_{i=1,\dots,N}(s_i)\underline{\sigma}(Q)\|e\|^2 \leq 0, \quad (38)$$

where $\underline{\sigma}(\cdot)$ is the minimum singular value. Since $\dot{V}_1 \leq 0$, this implies that the pair $(e, \tilde{\theta}) \in \mathcal{L}_\infty$ are bounded. From (38),

$$V_1\left(e(t \to \infty), \tilde{\theta}(t \to \infty)\right) = V_1\left(e(t_0), \tilde{\theta}(t_0)\right) - \min_{i=1,\dots,N}(s_i)\underline{\sigma}(Q)\|e\|^2 < \infty, \quad (39)$$

and establishes the limit on V_1 as $t \to \infty$. To show that $\ddot{V}_1$ is bounded, it is necessary to prove the boundedness of (26). By virtue of $\dot{V}_1 \leq 0$, then $e \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\tilde{\theta} \in \mathcal{L}_\infty$. Therefore, as θ is constant and bounded, this implies that the estimated value $\hat{\theta} \in \mathcal{L}_\infty$. $\underline{x}_0 = \underline{x}_{0,r}$ are bounded since the leader has zero input, then using the fact that $I_N \otimes A - c(L + G) \otimes BK$ is Hurwitz [19], it is shown that $(x_r, x, u_n) \in \mathcal{L}_\infty$. Accordingly, all terms on the right-hand side of (26) are bounded. This signifies that $\ddot{V}_1$ is bounded and $\dot{V}_1$ is a uniformly continuous function. By Barbalat's lemma [32], it can be said that $\dot{V}_1 \to 0$ and hence $e \to 0$ as $t \to \infty$. Therefore, the follower state is guaranteed to track the reference model.

4.1.2. Proof of $\delta \to 0$ as $t \to \infty$

Consider the following Lyapunov candidate function

$$V_2(\varepsilon_r) = \varepsilon_r^T(S \otimes P)\varepsilon_r. \quad (40)$$

Taking the first derivative of V_2 along (28),

$$\dot{V}_2 = \varepsilon_r^T[S \otimes (PA + A^T P) - c\{S(L + G) + (L + G)^T S\} \otimes PBR^{-1}B^T P]\varepsilon_r. \quad (41)$$

Using Lemma 1,

$$\dot{V}_2 = \varepsilon_r^T[S \otimes (PA + A^T P) - cT \otimes PBR^{-1}B^T P]\varepsilon_r. \quad (42)$$

Applying the same matrix property as in (35), then (42) can be represented as

$$\dot{V}_2 = \sum_{i=1}^{N} s_i\varepsilon_{i,r}^T \{PA + A^T P - cf_i\mu_i PBR^{-1}B^T P\}\varepsilon_{i,r}. \quad (43)$$

By choosing a coupling gain c that satisfies (29),

$$\dot{V}_2 \leq \sum_{i=1}^{N} s_i\varepsilon_{i,r}^T \{PA + A^T P - PBR^{-1}B^T P\}\varepsilon_{i,r} \quad (44)$$

or alternatively

$$\dot{V}_2 \leq - \min_{i=1,\dots,N}(s_i)\underline{\sigma}(Q)\|\varepsilon_r\|^2 \leq 0. \quad (45)$$

Let $\mathfrak{R}$ be the set of all points such that $\dot{V}_2 = 0$.

$$\mathfrak{R} = \{\varepsilon_r \in \mathcal{M}: \dot{V}_2(\varepsilon_r) = 0 \Rightarrow \varepsilon_r = 0\} \quad (46)$$

Thus, the invariant set $\mathcal{M} \subset \mathfrak{R}$ is the set that contains only the origin. Then according to LaSalle's invariant set theorem, $\varepsilon_r \to 0$ as $t \to \infty$. It signifies that the origin is asymptotically stable and implies that $\delta \to 0$ as $t \to \infty$. Therefore, the follower state is synchronized to the leader state.

This completes the proof. ∎

4.2. Vehicle Platoon with Undirected Topology

Theorem 2. *Consider a vehicle platoon with the dynamics expressed by (3) and (6). The network topology is assumed to be undirected, static, and connected, where at least one follower receives information from the leader. The reference model is constructed according to (7) and (8). By applying the distributed controller (13) with feedback gain K as in (10) and selecting the coupling gain c such that*

$$c \geq \frac{1}{2 \min_{i=1\cdots N}(\lambda_i)}, \quad (47)$$

and the adaptation law as

$$\dot{\hat{\theta}}_i = \gamma\lambda_i\Phi_i(x_i, u_{in})e_i^T PB, \quad (48)$$

where $\gamma > 0$ is the adaptation rate and λ_i is the ith eigenvalue of matrix $(L + G)$. Then the uncertain global vehicle state tracks the reference state, such that $e \to 0$ as $t \to \infty$, and the global tracking error satisfies $\delta \to 0$ as $t \to \infty$.

Proof. The following Lyapunov candidate functions are used to show that $e \to 0$ as $t \to \infty$ (V_3) and $\delta \to 0$ as $t \to \infty$ (V_4)

$$V_3(e, \tilde{\theta}) = e^T((L+G) \otimes P)e + \gamma^{-1}tr(\Omega^{1/2}\tilde{\theta}^T\tilde{\theta}\Omega^{1/2}), \quad (49)$$

and

$$V_4(\delta) = \delta^T((L+G) \otimes P)\delta. \quad (50)$$

Since the topology is undirected, by Lemma 2, $(L+G)$ is symmetric and positive definite. Let U be a unitary matrix such that $U^T(L+G)U = \Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_N)$. This property is used to derive the condition on the scalar coupling gain c. The rest of the stability analysis procedures are the same as Theorem 1 and will be omitted for brevity.

This completes the proof. ∎

Remark 4: Assume that the vehicle i is subjected to an unknown, bounded external disturbance $w_i \in \mathbb{R}^m$ such that

$$\dot{x}_i = Ax_i + B\Omega_i[u_i + \Omega_i^{-1}\eta_i(x_i) + \Omega_i^{-1}w_i] \quad (51)$$

where the global bound of the disturbances can be written as $\|\underline{w}\| \leq \omega$, $\omega > 0$, and $\underline{w} = col(w_1, w_2, \cdots, w_N) \in \mathbb{R}^{mN}$. By applying the proposed control (13), the global tracking error w.r.t the reference model (e) is uniformly ultimately bounded and satisfies $\lim_{t\to\infty}\|e\| \leq \alpha$ (for directed topology) and $\lim_{t\to\infty}\|e\| \leq \beta$ (for undirected topology), where

$$\alpha = \frac{2\max\limits_{i=1,..,N}(s_i)\bar{\sigma}(PB)\omega}{\min\limits_{i=1,..,N}(s_i)\underline{\sigma}(Q)} \quad (52)$$

and

$$\beta = \frac{2\max\limits_{i=1,..,N}(\lambda_i)\bar{\sigma}(PB)\omega}{\min\limits_{i=1,..,N}(\lambda_i)\underline{\sigma}(Q)}. \quad (53)$$

Here, $\bar{\sigma}(\cdot)$ is the maximum singular value. This implies that δ is also bounded by the same corresponding values.

5. Numerical Simulation

The performances and efficacy of the proposed DMRAC are analyzed by using a vehicle platoon with 1-leader and 3-followers, subjected to uncertain dynamics and unknown external disturbances. Two topologies are considered, namely BD to represent undirected topology and PF to represent directed topology, as shown in Fig. 3. The corresponding Laplacian and pinning gain matrices for both topologies are shown in Table 1.
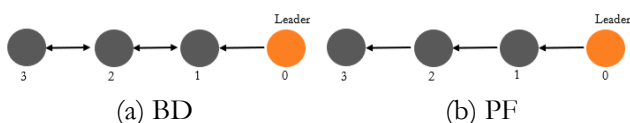

(a) BD   (b) PF

Fig. 3. The vehicle platoon topologies.

Table 1. The Laplacian and pinning gain matrices.

Matrices	Topologies	
	BD	PF
Laplacian Matrix, L	$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$
Pinning Gain Matrix, G	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Remark 5: BD and PF are chosen because they have the slowest responses compared to other undirected and directed topologies, respectively. In addition, the effects of local uncertainties and disturbances are propagated throughout the platoon [35].

A constant spacing policy is used with $d = 5\,m$. All vehicles involved in the platoon have a nominal inertial time lag of $\tau = 0.25\,s$. Therefore, the system matrices become

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}. \quad (54)$$

The control effectiveness and the constant weight matrices for each vehicle are represented in Table 2.

Table 2. The control effectiveness and constant weight matrices of follower vehicles.

Vehicle (i)	Ω_i	W_i^T
1	0.4	[0 0 -1.5]
2	0.5	[0 0 0.375]
3	0.5	[0 0 -0.67]

The unknown, external disturbances acting on each vehicle are specified as

$$\begin{cases} w_1 = 0.5\cos(0.5\pi t)\sin(0.3\pi t), \\ w_2 = 2 + \sin(0.5\pi t), \\ w_3 = 2.5\sin(0.3\pi t). \end{cases} \quad (55)$$

Remark 6: External disturbances can result from traffic conditions, windy roads, aerodynamic drag, or parameter variations [16]. Sinusoidal-type disturbances similar to (55) have been used to verify controllers for vehicle platoons in [36, 37].

The initial position, velocity and acceleration of all vehicles are shown in Table 3.

Table 3.   Initial conditions of all vehicles.

Vehicles (i)	$p_i(0)$ [m]	$v_i(0)$ [m/s]	$a_i(0)$ [m/s²]
0	45	20	0
1	35	18	0
2	20	22	0
3	8	24	0

The nominal controller is designed using LQR according to (13), with $Q = diag\{1,1,1\}$ and $R = 0.1$, resulting in the matrix P and feedback gain K as follows,

$$P = \begin{bmatrix} 1.8324 & 1.1789 & 0.0791 \\ 1.1789 & 2.0811 & 0.1449 \\ 0.0791 & 0.1449 & 0.0682 \end{bmatrix}, \quad (56)$$

$$K = [3.1623 \quad 5.7946 \quad 2.7279]. \quad (57)$$

The coupling gain and adaptation rate for a vehicle platoon with BD are $c = 1.3$ and $\gamma = 0.1$. For a vehicle platoon with PF, the coupling gain and adaptation rate are $c = 2.45$ and $\gamma = 0.01$.

Remark 7: The choices of Q and R reflect the trade-off in optimal LQR controller designs between tracking performance and control input [38]. Similarly, increasing the coupling gain c improves the synchronization performance of the platoon at the cost of a large initial control effort. The adaptation rate γ determines how fast the followers can track to reference model but may result in high frequency oscillation of the control signal [32].

5.1. Vehicle Platoon with Undirected Topology

The objective of this section is to show the effectiveness of the proposed DMRAC for a vehicle platoon with undirected topology (BD). Firstly, it is assumed that the vehicle platoon contains uncertainties in control effectiveness and constant weight matrices, as shown in Table 2. Numerical simulation demonstrates that the uncertain vehicles can track the reference model, as shown in Fig. 4, with the corresponding uncertainty estimation error, $\tilde{\theta}_i^T \Phi_i(x_i, u_{in})$, shown in Fig. 5.

Synchronization to the leader state is shown in Fig. 6 and compared to conventional cooperative state variable feedback (CSVFB). Performance improvements in the tracking error of state positions are clearly observed and listed in Table 4.

Moreover, when each of the followers experience unknown external disturbances according to (55), then the system with DMRAC is still able to achieve synchronization to the leader state with small, bounded residual errors as shown in Fig. 7 and Table 5.
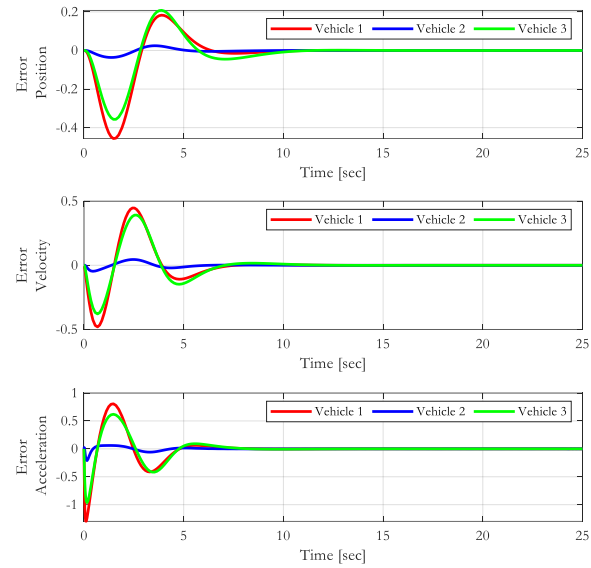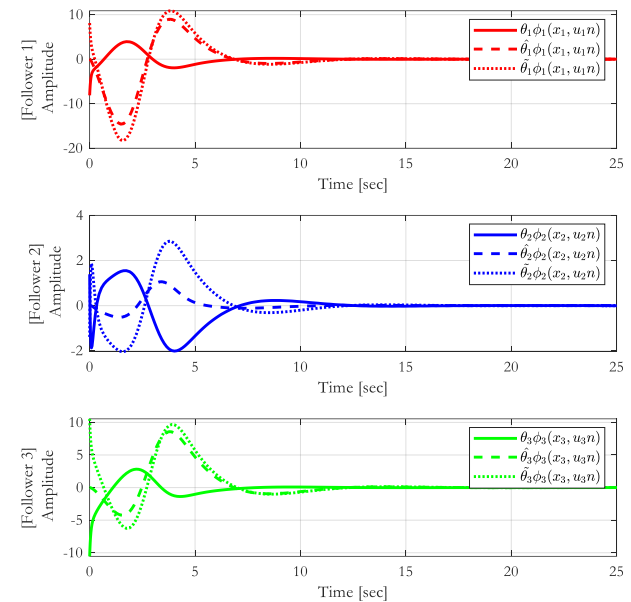

Fig. 4. Tracking error (e_i) w.r.t the reference model (BD).

Fig. 5. The uncertainty estimation (BD).

Table 4.   Performance comparison, DMRAC vs CSVFB.

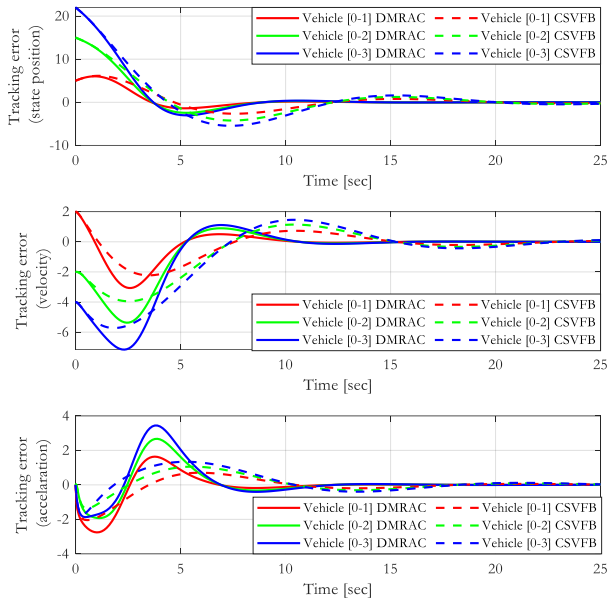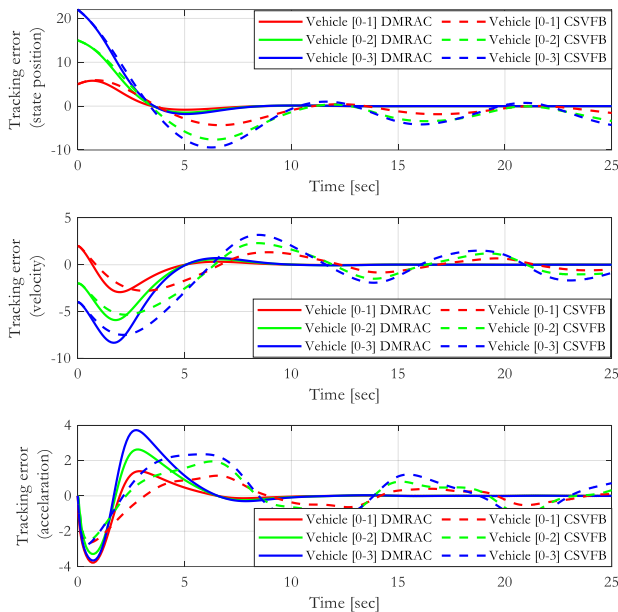| Vehicle(i)	Performance Specification	DMRAC	CSVFB
Vehicle 1	Settling time	9 s	20 s
	Overshoot	21.4%	34.6%
	Peak time	5 s	7.5 s
	Rise time	3.6 s	4.7 s
Vehicle 2	Settling time	9 s	20 s
	Overshoot	13.5%	21.9%
	Peak time	5 s	7.5 s
	Rise time	3.6 s	4.7 s
Vehicle 3	Settling time	9 s	20 s
	Overshoot	11.6%	19.8%
	Peak time	5 s	7.5 s
	Rise time	3.6 s	4.7 s

Fig. 6. Tracking error (δ_i) w.r.t the leader (BD).

Fig. 7. Tracking error (δ_i) w.r.t the leader when followers experience bounded disturbances.

Table 5. Bounded error comparison, DMRAC vs CSVFB.

Error	DMRAC		CSVFB	
$t > 15\,s$	Min	Max	Min	Max
Distance[m]	-0.009	0.006	-4.31	0.74
Velocity[m/s]	-0.008	0.010	-1.68	1.51
Acceleration[m/s²]	-0.010	0.012	-1.33	1.21

5.2. Vehicle Platoon with Directed Topology

The objective of this section is to show the effectiveness of the proposed DMRAC for a vehicle platoon with directed topology (PF). When the vehicles are subjected to uncertainties in control effectiveness and constant weight matrices, DMRAC ensures that the vehicle states continue to track the reference model, as shown by the tracking error in Fig. 8. The corresponding uncertainty estimation error is shown in Fig. 9. The synchronization performance with the leader state and comparison to CSVFB are shown in Fig. 10 and Table 6.
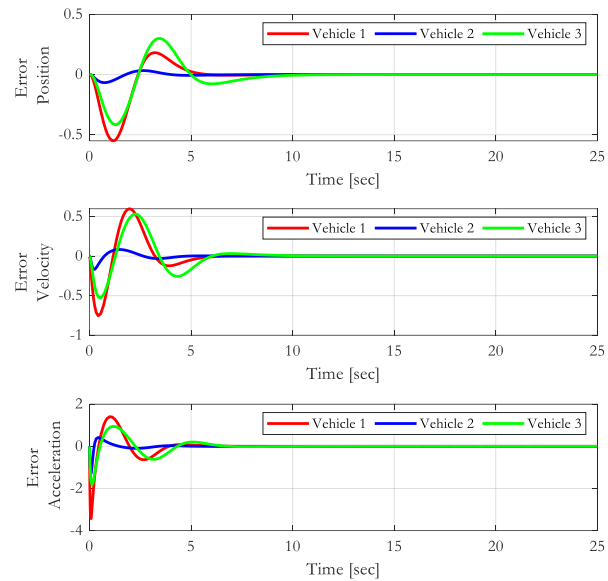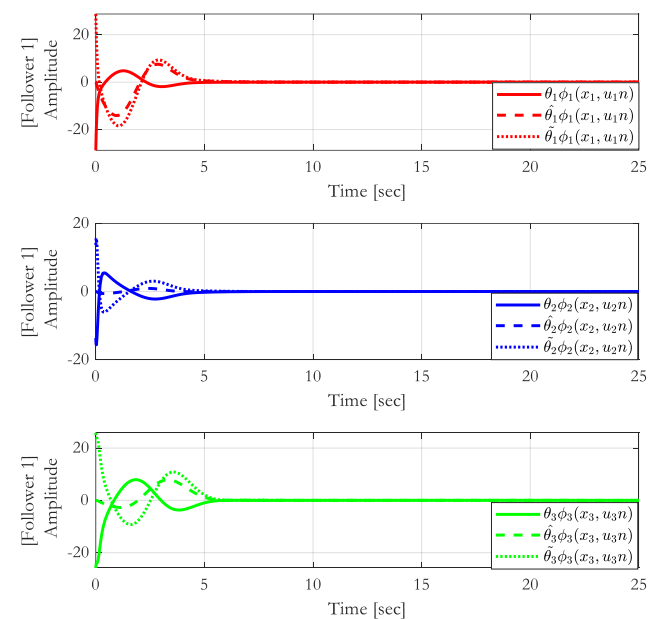

Fig. 8. Tracking error (e_i) w.r.t the reference model (PF).

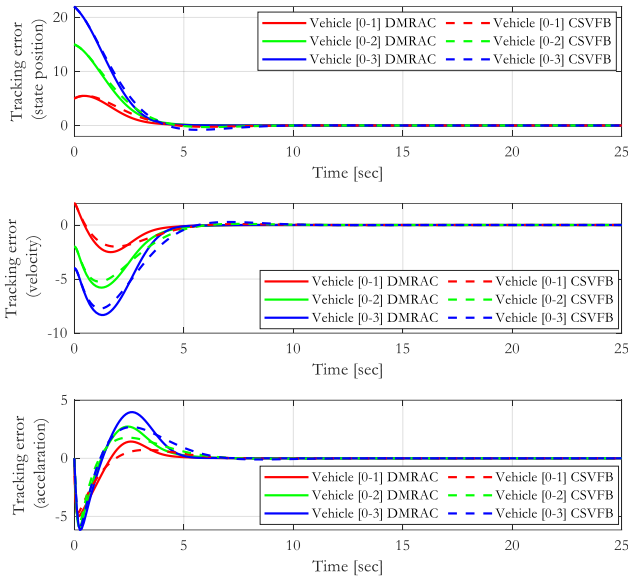Fig. 9. The uncertainty estimation (PF).

Fig. 10. Tracking error (δ_i) w.r.t the leader (PF).

Table 6.   Performance comparison, DMRAC vs CSVFB.

Vehicle(i)	Performance Specification	DMRAC	CSVFB
Vehicle 1	Settling time	5 s	9 s
	Overshoot	0 %	3.9 %
Vehicle 2	Settling time	5 s	9 s
	Overshoot	0 %	1.1 %
Vehicle 3	Settling time	5 s	9 s
	Overshoot	0 %	1.1 %

The efficacy of the proposed DMRAC is also shown when the vehicles experience unknown external disturbances, as shown in Fig. 11. The analysis of the bounded residual error is listed in Table 7.
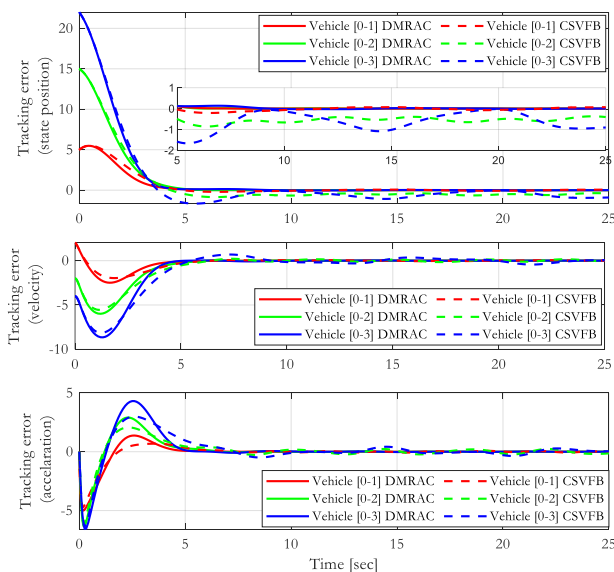

Fig. 11. Tracking error (δ_i) w.r.t the leader, PF with disturbance.

Table 7. Bounded error comparison, DMRAC vs CSVFB.

Error	DMRAC		CSVFB	
$t > 15\,s$	Min	Max	Min	Max
Distance[m]	-0.014	0.023	-1.00	0.07
Velocity[m/s]	-0.012	0.015	-0.44	0.36
Acceleration[m/s²]	-0.028	0.019	-0.36	0.31

6.   Conclusion

This paper presents a distributed model reference adaptive control to overcome the problems of a vehicle platoon with uncertainties in control effectiveness and inertial time lag. The controller can be applied to any vehicle platoon with both directed and undirected topologies. The directed topology must contain at least one spanning tree with the leader as a root node, while the undirected topology must be static and connected with at least one follower receiving information from the leader. The conditions on the coupling gain and adaptation law to ensure stability is derived. Through stability analysis and simulation, it is shown that the uncertain vehicles can track the reference model and achieve synchronization to the leader state. When the followers experience disturbances, the global tracking error remains uniformly ultimately bounded and the vehicles can synchronize to the leader state with small residual errors.

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Agung Prayitno was born in Klaten, Central Java, Indonesia in 1979. He received the B.Eng. degree in Electrical Engineering from Sepuluh Nopember Institute of Technology (ITS), Surabaya, Indonesia in 2003. He received the Master degree in Control System Engineering in Hogeschool van Arnhem en Nijmegen, Netherlands in 2010. He is currently pursuing a Ph.D. degree with the School of Information, Computer, and Communication Technology, Sirindhorn International Institute of Technology, Thammasat University, Thailand.

His research interests include cooperative control of multi-agent system, model reference adaptive control, and optimal control.

Itthisek Nilkhamhang received his B.Eng. in Electrical Engineering from Sirindhorn International Institute of Technology, Thammasat University, in 2002, and his M.Eng. and Ph.D. in Integrated Design Engineering from Keio University in 2005 and 2008, respectively. He joined Sirindhorn International Institute of Technology, Thammasat University, as a full-time faculty member in 2008. He is currently an assistant professor in the School of Information, Computers, and Communications Technology, as a member of the Electrical Engineering Program.

His research interests include iterative tuning and learning algorithms, robust and adaptive control systems, system identification, robotics, mechatronics, instrumentation and automation.