Engineering Journal issn: 0125-8281

Article

Convex Optimization Approach to Multi-Objective Design of Two-Stage Compensators for Linear Systems

Tanathorn Supithak[1], **Nicha Wongyimyong**[1], **Kou Yamada**[2,a], **and David Banjerdpongchai**[1,b,*]

1 Intelligent Control Automation of Process Systems Research Unit, Department of Electrical Engineering, Faculty of Engineering, Chulalongkorn University, 254 Phayathai Road, Phathumwan, Bangkok, 10330, Thailand
2 Graduate School of Science and Technology, Gunma University, 1-5-1 Tenjincho, Kiryu 376-8515, Japan
E-mail: [a]yamada@gunma-u.ac.jp, [b,*]bdavid@chula.ac.th (Corresponding author)

Abstract. The previous design of two-stage compensators of linear systems was focused on the stabilization and low sensitivity. However, it has not considered the time-domain performance of the closed-loop system, especially, reference tracking. This paper aims to propose the design method of the two-stage compensators that additionally achieves good transient response. Applying Q-parameterization to the standard control system can formulate the two-stage compensator design as a convex optimization problem. The infinite dimensional problem is transformed into a finite dimensional problem by Ritz approximation. Finally, the convex optimization is efficiently solved to give the optimal controller. The numerical results show that the proposed design method on the second order benchmark problem and the first order plus time delay system improves the time-domain performance while the closed-loop system is stable and the influence of disturbance to output is attenuated.

Keywords: Two-stage compensators, multi-objective criteria, Q-parameterization, convex optimization, reference tracking, low sensitivity.

ENGINEERING JOURNAL Volume 25 Issue 8
Received 11 June 2021
Accepted 10 August 2021
Published 31 August 2021
Online at https://engj.org/
DOI:10.4186/ej.2021.25.8.153

This article is based on the presentation at The 2021 2nd International Symposium on Instrumentation, Control, Artificial Intelligence, and Robotics (ICA-SYMP 2021) in Bangkok, Thailand, 20th-22nd January 2021.

DOI:10.4186/ej.2021.25.8.153

1. Introduction

In the control system, there are many kinds of controllers and compensators that can be applied to manipulate the system behavior. Choosing the control method based on the system mathematical model and designing the controller properly can make the system to achieve its design objectives.

The design of two stage compensator can be done using the factorization approach [1, 2, 3], Youla-parameterization, and the method to find a proper stabilizing controller that satisfies the specified design criteria [4, 5]. Since the method describes controllers into a single parameter Q, Youla-parameterization is also known as Q-parameterization. This Q parameter leads to the controller $K(Q)$ which can stabilize the given linear time-invariant plant. The Q parameterization method can be implemented in three main configurations, including controller parameterization, plant parameterization, and simultaneous control and plant parameterization. Controller parameterization is mainly used in stabilizing system and disturbance rejection control. While plant parameterization can identify the closed-loop system. Lastly, simultaneous control and plant parameterization is appropriate for providing a new control structure that changes depending on reformation of the dynamics on the plant. Many researches introduce Q parameterization to conduct the design solution. For example, [6] proposed Youla parameterization for the plant obtained from H_∞ norm and a central controller. While the authors of [7] introduce adaptive controller for fault tolerant control (FTC) using Youla parameterization method. It is noted that [8] presents the design of stabilizing controllers subject to quadratically invariant (QI) subspace constraint. Since the Youla parameter Q has infinite-dimension, Ritz approximation is also frequently used with Q parameterization to construct the finite-dimension optimization problem for optimization [9]. Some of the research studies [10, 11, 12] established these combined methods.

The design of two-stage compensator to make the closed-loop system achieves the design objectives including stability and low sensitivity has been considered in [13, 14]. Given that the first compensator is stable, the second compensator is not necessary to be stable. The design aims to stabilize the closed-loop system. Furthermore, according to the sensitivity function of the closed-loop system, the size of noise is reduced at low frequency range. In other words, a design method of two-stage compensator systems aimed to attenuate the steady state error for the reference input and the influence of the disturbance for the output. In addition, the authors of [13] showed a design method that achieves low sensitivity characteristics and maintains the stability of control system. Nonetheless, the previous research has not addressed design of two-stage compensators that achieve good time-domain performances including reference tracking with specified transient response. Deign of compensators including these conditions can make the closed-loop system more efficient.

This research work proposes the design of two-stage compensators with multi-objective criteria including low sensitivity and good reference tracking, especially, the transient response of the closed-loop system. By using Q-parametrization and Ritz approximation, the compensator design is transformed to a convex optimization problem which can be efficiently solved by available solvers. We demonstrate the simulation results of two linear systems and show the performance of the closed-loop systems. The design method renders stable closed-loop systems with low sensitivity and satisfactory transient response.

The paper is organized as follows. Section 2 describes the problem statement, followed by design via convex optimization in Section 3. In Section 4, numerical results are illustrated the effectiveness of the proposed design. Conclusions are given in the Section 5.

2. Problem Statement

From [13, 14], the design of two-stage compensators is based on the control structure shown in Fig. 1.

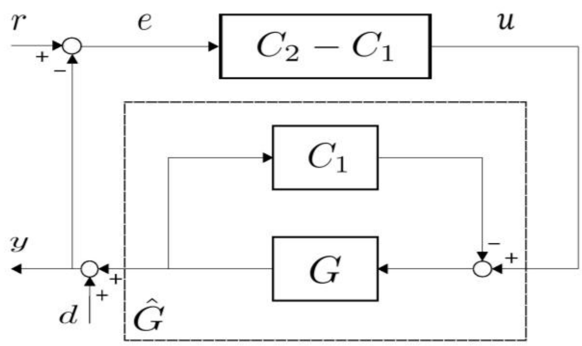

Fig. 1. System with two-stage compensators.

G is the transfer function of plant, C_1 is the first compensator, C_2 is the second compensator, r is the input signal, d is the disturbance signal, y is the output signal, e is the error between output signal and reference signal, and u is the control signal.

We design the first compensator C_1 to ensure stability of the inner loop. Many design techniques could be used. Some simple systems require only unity feedback $C_1 = 1$. Complex systems may require PID controllers. The closed-loop transfer function can be written as

$$\hat{G}(s) = \frac{G}{1+C_1 G} \tag{1}$$

In this section, we will consider the design of the second compensator to make the system achieves the design objectives in terms of low sensitivity and reference tracking performance. The design approach is based on Q-parameterization and Ritz approximation which leads to control design using convex optimization.

2.1. Standard Form of Control System

The system can be rewritten as shown in fig. 2. Hence, the closed-loop transfer function and sensitivity transfer function are defined in Eq. (2) and (3), respectively.

$$T = \frac{y}{r} = \frac{(C_2 - C_1)\hat{G}}{1+(C_2-C_1)\hat{G}} \tag{2}$$

$$S = \frac{y}{d} = \frac{e}{r} = \frac{1}{1+(C_2-C_1)\hat{G}} \tag{3}$$

From the Eq. (2) and (3), we can see that the relationship between closed-loop transfer function and sensitivity function can be written as

$$S = 1 - T \tag{4}$$

In our study, the design of compensator ensures that the closed-loop system in low frequency range has a gain of T approximately equal to 1. It is equivalent to show that the sensitivity function has small magnitude in the low frequency. Hence, the closed-loop system with two-stage compensators can eliminate the effect of disturbance.

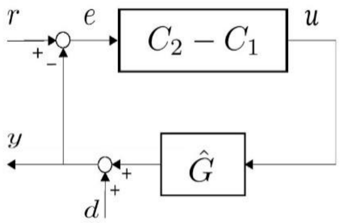

Fig. 2. The equivalent system.

This research applies the standard form of the closed-loop system being described in [15]. Figure 3 demonstrates the block diagram of the standard form.

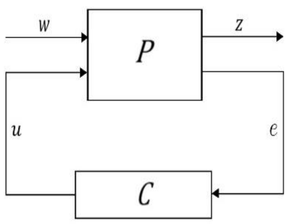

Fig. 3. Standard form of closed-loop control system.

By considering w and u as input and z and e as output, we can write P as transfer matrix as Eq. (5).

$$P = \begin{bmatrix} P_{zw} & P_{zu} \\ P_{ew} & P_{eu} \end{bmatrix} \tag{5}$$

By comparing to the system with two-stage compensator, which is shown in the block diagram of Figure 3, we obtain the actual matrix P in this form

$$P = \begin{bmatrix} 0 & 1 & \hat{G} \\ 0 & 0 & 1 \\ 1 & -1 & -\hat{G} \end{bmatrix} \tag{6}$$

Let H_{zw} be transfer matrix from input w to output z, we have

$$H_{zw} = P_{zw} + P_{zu}(C_2 - C_1)(1 - P_{eu}(C_2 - C_1))^{-1} P_{ew} \tag{7}$$

Now, assume that $C_2 - C_1$ has no feedback data ($P_{eu} = 0$), Eq. (7) is reduced to

$$H_{zw} = P_{zw} + P_{zu}(C_2 - C_1)\, P_{ew} \tag{8}$$

It can be seen that, H_{zw} is related to $C_2 - C_1$. Therefore, the system with two-stage compensator is rewritten to the standard form and two-stage compensator design problem is set up.

2.2. Q-Parameterization

After system is written in the standard form, by adapting Q-Parameterization method, parameter Q which makes the closed-loop system achieves the stability objective can be found by the following steps [1].
First, let

$$\hat{G} = \frac{N}{D} \tag{9}$$

Then, choose X and Y satisfying

$$XN + YD = 1 \tag{10}$$

so that we can obtain $C_2 - C_1$ as a function of Q which will stabilize $\hat{G}$ as follows

$$C_2 - C_1 = \frac{X + DQ}{Y - NQ} \tag{11}$$

where $N, D, X, Y,\ Q$ are proper and stable transfer functions.

According to [15] and [16], H_{zw} can be rewritten as a function of Q as follows.

$$H_{zw} = T_1 + T_2 Q T_3 \tag{12}$$

where

$$T_1 = P_{zw} + P_{zu} D X P_{ew}$$
$$T_2 = P_{zu} D$$
$$T_3 = D P_{ew}$$

Thus, we consider H_{zw} which is a convex function of Q and the design problem of two-stage compensator can be transformed to convex optimization problem of Q.

2.3. Ritz Approximation

Since Q can be freely designed, the dimension of Q is infinite. Using Ritz approximation employs finite dimensional Q [15, 16]. By letting

$$H_{zw} = R_0 + \sum_{1 \le i \le N} x_i R_i \tag{13}$$

where $x = [x_1\ x_2 \cdots x_{\mathrm{N}}]^T \in \mathrm{R}^{\mathrm{N}}$, N is the dimension of Q which is equivalent to the dimension of x, and $R_0, R_1, \dots, R_{\mathrm{N}}$ are arbitrary transfer functions. By comparing Eq. (12) and (13), $R_0, R_1, \dots, R_{\mathrm{N}}$ can be properly chosen as follows.

$$R_0 = T_1,\ R_i = T_2 Q_i T_3, i = 1,\ 2,\ \dots, \mathrm{N}$$

Then, Q will be a linear combination of $Q_1, Q_2, \dots, Q_{\mathrm{N}}$ as follows.

$$Q = x_1 Q_1 + x_2 Q_2 + \cdots + x_{\mathrm{N}} Q_{\mathrm{N}} \tag{14}$$

where $Q_1, Q_2, \dots, Q_{\mathrm{N}}$ are arbitrary transfer functions.

From the method mentioned above, the convex optimization problem of Q is changed to convex optimization problem of x, which has a finite dimension.

The problem statement is to design the compensator for the control system which guarantees the stability, achieves low sensitivity characteristics, and satisfies the time-domain performance for the reference tracking.

3. Design via Convex Optimization

In this section, we want to design the compensator such that the closed-loop system achieves the objective of transient response of the step reference tracking. The performance measures are rise rime (t_r), settling time (t_s), and maximum overshoot (M_p). Rise time is defined as time period that response increases from 0 to 80% of the steady state value. Settling time is time of the response reaching the final value with ±2% bound, and the maximum overshoot is the maximum error between the reference signal and the output signal, which is measured in percentage of steady state value.

Now these performance measures will be used to create convex optimization problem. A convex optimization problem of Q which has infinite dimension is transformed to a convex optimization problem of x which has finite dimension by using Ritz approximation. Moreover, the transfer function of the closed-loop system and the sensitivity can be written as function of x. We define $s(x,t)$ as an output response to a unit step input. Settling time, rise time, and maximum overshoot can be written in form of $s(x,t)$. These performance indices are convex functions. Thus, optimizing x for these performance measures can be found by solving the convex optimization. The time-domain specifications can be written as follows [15].

Maximum overshoot: $\max(s(x,n)) - 1$
Rise time: $\inf\{n_r | s(x,n) > 0.8 \text{ for } n \geq n_r\}$;
Settling time: $\inf\{n_r | |s(x,n) - 1| < 0.02 \text{ for } n \geq n_s\}$

where $n_r = \frac{t_r}{T_{samp}} + 1$, $n_s = \frac{t_s}{T_{samp}} + 1$, T_{samp} is a sampling time.

Now, we can set one performance measure as an objective function and the other measures are constraint functions. To illustrate the design criteria, the first possible problem is as follows.

Minimize M_p
subject to $t_r \leq \alpha$ and $t_s \leq \beta$.

Alternatively, the second possible problem is as follows.

Minimize t_r
subject to $M_p \leq \alpha$ and $t_s \leq \beta$.

Lastly, the third possible problem is as follows.

Minimize t_s
subject to $M_p \leq \alpha$ and $t_r \leq \beta$.

Note that α and β are constraint parameters of the performance measures. Finally, the convex optimization problem can be solved by using the available solvers.

In this work, CVX [17, 18] is chosen as a solver for convex optimization and MATLAB is employed to compute the output response to a unit step input. Moreover, we display the closed-loop performance to show that system using the two-stage compensator reaches the design objective, which are closed-loop stability, low sensitivity, and satisfactory time-domain performance.

4. Numerical Results

4.1. Design of Two-stage Compensators for the Second Order System

To apply the methodology, we employ a 2nd-order system as in Eq. (15), and the first compensator C_1 as in Eq. (16).

$$G = \frac{{\omega_n}^2}{s(s+2\zeta\omega_n)} \tag{15}$$

$$C_1 = 1 \tag{16}$$

By substituting Eq. (15) and (16) into Eq. (1), we have

$$\hat{G} = \frac{{\omega_n}^2}{s^2+2\zeta\omega_n s+{\omega_n}^2} \tag{17}$$

From Eq. (9) and (17) N and D can be defined as follows.

$$N = \frac{{\omega_n}^2}{s^2+2\zeta\omega_n s+{\omega_n}^2} \tag{18}$$

$$D = 1 \tag{19}$$

Next, choose X and Y that corresponds to Eq. (10) as follows

$$X = 1 \tag{20}$$

$$Y = \frac{s^2+2\zeta\omega_n s}{s^2+2\zeta\omega_n s+{\omega_n}^2} \tag{21}$$

Finally, $C_2 - C_1$ and H_{zw} can be rewritten in form of Q as shown in Eq. (11) and (12), respectively. Therefore, we formulate the compensator design as the convex optimization of Q. From Eq. (4), (6), (9), (12), (19), and (20), we obtain

$$H_{zw} = \begin{bmatrix} N(1+Q) & 1-N(1+Q) \\ 1+Q & -(1+Q) \end{bmatrix} \tag{22}$$

where $w = \begin{bmatrix} r \\ d \end{bmatrix}$ and $z = \begin{bmatrix} y \\ u \end{bmatrix}$. By considering Eq. (22), the transfer function of the closed-loop system and the sensitivity can be written as follows

$$T = N(1+Q) \tag{23}$$
$$S = 1 - N(1+Q) \tag{24}$$

respectively. Since the closed-loop transfer function and the sensitivity transfer function are related with Q, selecting proper $Q_i, i = 1, 2, \ldots, \mathrm{N}$, shall make the closed-loop system stable, has low sensitivity, and meets the time-domain performance criteria.

The proper Q_i should have all poles in the LHP to make closed-loop system stable and should have zero at the origin for low sensitivity and steady-state error. In particular, we choose Q_i as

$$Q_i = \frac{s}{(s+a)^i} \tag{25}$$

where $a \in \mathrm{R}^+,\ i = 1, 2, \ldots, \mathrm{N}$. Substituting (25) into Eq. (14), Q can be obtained as linear combination of $Q_1, Q_2, \ldots, Q_N$ as follows,

$$Q = \frac{x_1 s}{s+a} + \frac{x_2 s}{(s+a)^2} + \cdots + \frac{x_\mathrm{N} s}{(s+a)^\mathrm{N}} \tag{26}$$

Then, we substitute Eq. (18) and (26) into Eq. (23), the transfer function of the closed-loop system is now in form of

$$T = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}\left(1 + \frac{x_1 s}{s+a} + \cdots + \frac{x_N s}{(s+a)^N}\right)$$

As a result, all poles are in LHP, so the objective of stability is achieved. When we substitute Eq. (18) and (26) into Eq. (24), the transfer function of sensitivity can be written in a form of

$$S = \frac{s^2 + 2\zeta\omega_n s}{s^2 + 2\zeta\omega_n s + {\omega_n}^2} - \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}\left(\frac{x_1 s}{s+a} + \cdots + \frac{x_N s}{(s+a)^N}\right)$$

The transfer function of the sensitivity has zeros at the origin. Hence, the closed-loop system achieves its objective of low sensitivity and steady-state error.

Next, we will show the numerical results. Let the open-loop system be

$$G_1 = \frac{4}{s\,(s+2)}$$

and let

$$C_1 = 1$$

We select Q_i in the form of

$$Q_i = \frac{s}{(s+3)^i},\ i = 1, 2, \ldots, \mathrm{N}. \tag{27}$$

In this example, the maximum overshoot is selected as the objective function and the settling time and the rise time are constraints with specified bounds.

$$\alpha_1 = 1.425, \beta_1 = 4.300$$

As a result, we formulate the compensator design as the convex optimization problem.

minimize M_p
subject to $t_r \leq 1.425$ and $t_s \leq 4.300$.

We implement the convex optimization problem using CVX [17].

```
minimize max(s(x,n))-1
subject to
zeros(261,1)<= s(x,n)(1:261)<=0.8*ones(261,1);
s(x,n)(861)>=0.98;
```

From Q_i in the Eq. (27), let a be equal to 3 and vary N for 3 values as shown in Table 1. By solving the convex optimization problem of x and substituting x into Eq. (14), Q can be obtained as linear combination of $Q_1, Q_2, \ldots, Q_\mathrm{N}$ as in Table 2.

Table 1. Choosing N and $\boldsymbol{Q_i}$.

Design	N	$\boldsymbol{Q_i}$
1	1	$\frac{s}{(s+3)^i}, i = 1$
2	2	$\frac{s}{(s+3)^i}, i = 1, 2$
3	3	$\frac{s}{(s+3)^i}, i = 1, 2, 3$

Table 2. Design parameters x *and* Q.

N	Optimal value of x	Q
1	$x_1 = -1.005$	$-\frac{1.005s}{s+3}$
2	$x_1 = 1.264$ $x_2 = -6.439$	$\frac{1.264s}{s+3} - \frac{6.439s}{(s+3)^2}$
3	$x_1 = -1$ $x_2 = 3.764$ $x_3 = -15.743$	$-\frac{s}{s+3} + \frac{3.746s}{(s+3)^2} - \frac{15.743s}{(s+3)^2}$

Finally, the result confirms that the designed two-stage compensator can achieve the multi-objectives. The closed-loop system is stable, as shown in transient response to step input in Fig. 4 and the output response converges to the steady state value. The control input is shown in Fig. 5. The closed-loop system has zero steady-state error. The output response properly tracks the step input. The feedback system has low sensitivity as shown in Fig. 6. The Bode-plot of the sensitivity function in low frequency range clearly shows that the noise attenuation is reduced.

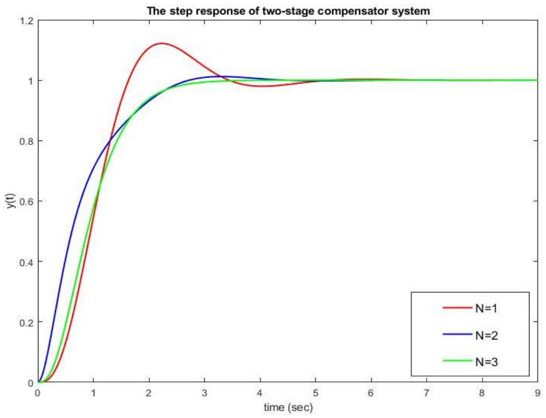

Fig. 4. Transient response to step input of the closed-loop system.

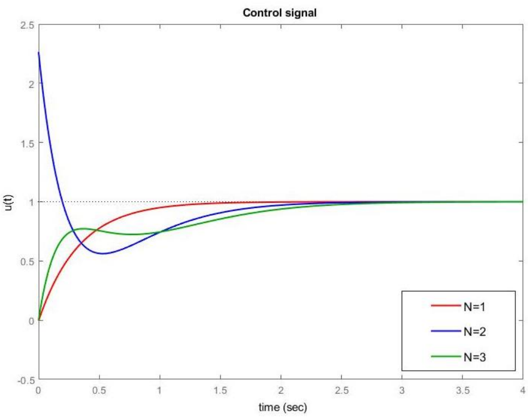

Fig. 5. Control signal of the closed-loop system.

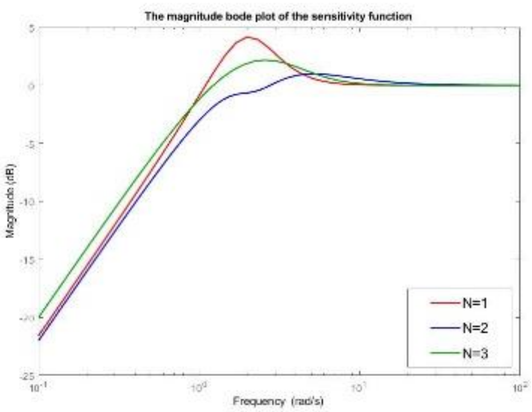

Fig. 6. Frequency response of the sensitivity function.

Table 3. Time domain performance of the first example.

Design	Performance Index		
	Rise time (s)	Settling time (s)	Maximum overshoot (%)
Specification	1.425	4.300	Objective function
N=1	1.300	3.215	12.172
N=2	1.300	2.425	1.209
N=3	1.425	2.505	0

Transient response has good performance in terms of the settling time, the rise time, and maximum overshoot. Table 3 shows the result that the rise time and settling time meet the specified constraints and the maximum overshoot is improved when the order of Q is increased and achieve zero maximum overshoot when the order of Q is 3. Using the mathematical model of the second-order system in Eq. (15), and the first compensator (C_1) in Eq. (16). The result is consistent to the former results in [13], which the second compensator is not necessary to be stable. From the designed two-stage compensator, considering the second compensator C_2 shown in Table 4, the higher dimension of Ritz approximation results in the higher order of the second compensator C_2.

Table 4. The designed 2nd compensator (C_2) for the first example.

N	The 2nd compensator (C_2)
1	$C_2 = \dfrac{4.02s^3+32.04s^2+64s+48}{4s^3+20s^2+39.92s}$
2	$C_2 = \dfrac{13.06s^4+63.52s^3+162.82s^2+240s+144}{4s^4+32s^3+63.78s^2+114.35s}$
3	$C_2 = \dfrac{4s^5+70.98s^4+303.94s^3+683.96s^2+864s+432}{4s^5+44s^4+196s^3+360.08s^2+432.08s}$

4.2. Design of Two-stage Compensators for the First Order plus Time Delay System

Another example system that can be designed with two-stage compensators to is the first order plus time delay (FOPTD) system. According to [19, 20, 21], applying PI controller to systems with time delay can guarantee robust stability. Hence, we will modify the first compensator C_1 with a PI controller to make system stable, as shown in Fig. 7, then the following steps of designing C_2 via convex optimization are unchanged.

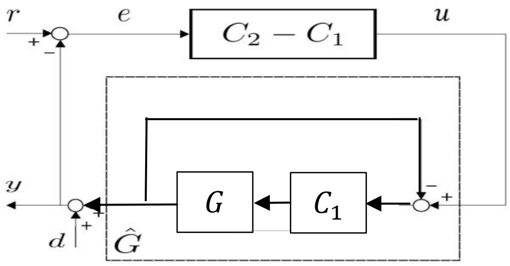

Fig. 7. Two-stage compensator system with modified C_1 as PI Controller.

Consider the FOPDT system

$$G(s) = \frac{1.479e^{-0.5s}}{0.2015s+1} \tag{28}$$

The PI controller used to control G is displayed as follows.

$$C_1 = 0.2 + \frac{1}{s} \tag{29}$$

The transfer function of the inner closed-loop $(\hat{G})$ is

$$\hat{G}(s) = \frac{GC_1}{1+GC_1} = \frac{1.479(0.2s+1)e^{-0.5s}}{0.2015s^2+s+1.479(0.2s+1)e^{-0.5s}} \tag{30}$$

Next, by using the same Q_i as in Eq. (27) with $a = 3$, transfer function of the system using two-stage compensators can be rewritten in this form.

$$T = \hat{G}(s)(1 + \frac{x_1 s}{s+a} + \cdots + \frac{x_N s}{(s+a)^N})$$

As a result, the convex optimization problem of parameter x is formulated and can be solved with CVX via MATLAB. Maximum overshoot is selected as an objective function while rise time and settling time are constraints.

minimize M_p
subject to $t_r \leq 1.425$ and $t_s \leq 4.300$.

We vary the dimension of Ritz approximation for 3 values. Table 5 shows the result x from solving the problem.

Table 5. Design parameters x and $\boldsymbol{Q}$ for FOPTD system.

N	Optimal value of $\boldsymbol{x}$	$\boldsymbol{Q}$
1	$x_1 = 0.1641$	$\frac{0.1641s}{s+3}$
2	$x_1 = 1.3029$ $x_2 = -4.5836$	$\frac{1.3029s}{s+3} - \frac{4.5836s}{(s+3)^2}$
3	$x_1 = 1.4640$ $x_2 = -7.9821$ $x_3 = 7.8103$	$\frac{1.4640s}{s+3} - \frac{7.9821s}{(s+3)^2} + \frac{7.8103s}{(s+3)^2}$

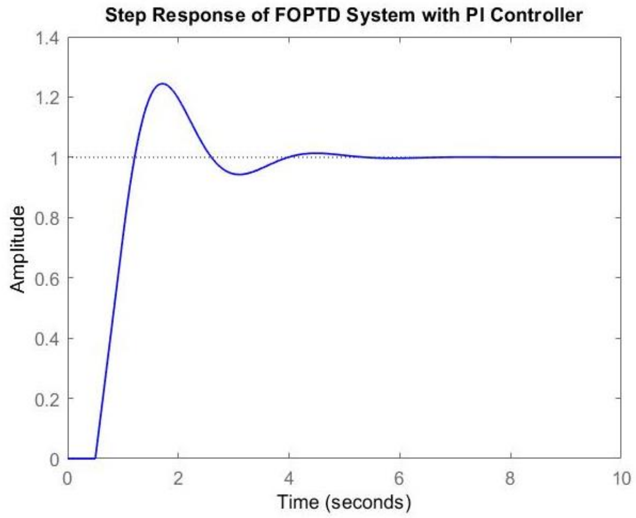

Fig. 8. Step response of the FOPTD system with PI controller.

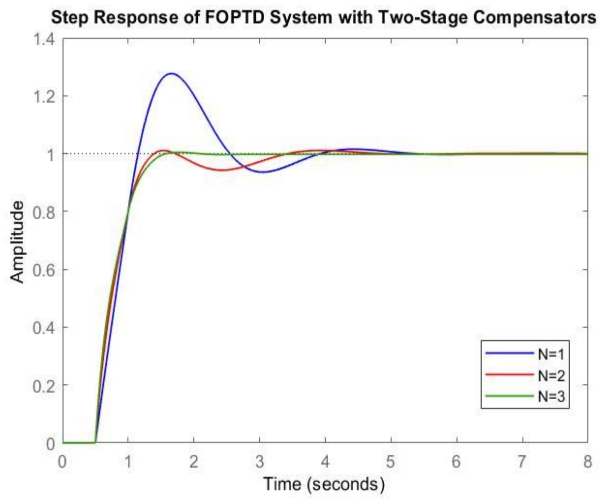

Fig. 9. Transient response of the closed-loop FOPTD system to a unit step input.

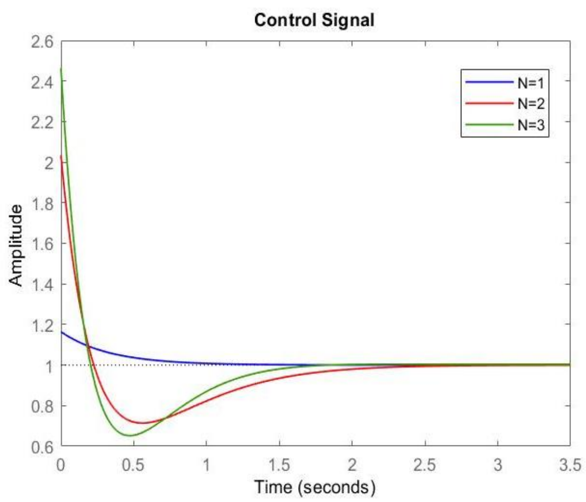

Fig. 10. Control signal of the closed-loop FOPTD system.

DOI:10.4186/ej.2021.25.8.153

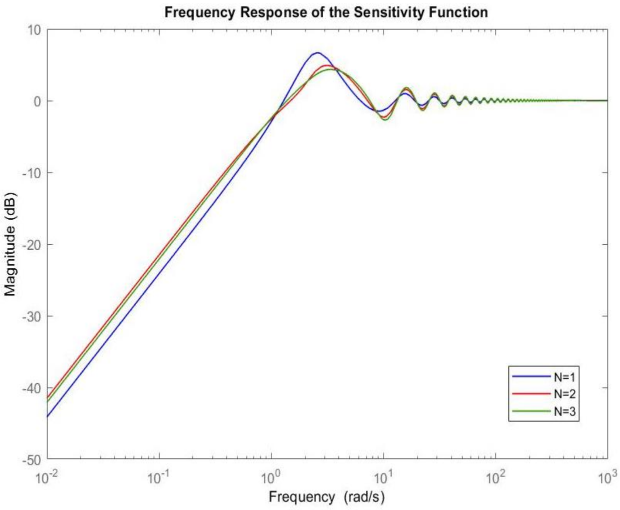

Fig. 11. Frequency response of the sensitivity function for the FOPTD system.

Figure 8 displays the step response of the FOPTD system with PI controller (i.e., step response of transfer function $\hat{G}$). It shows that the PI controller can make system stable. The delay time of 0.5 seconds in the original plant also cause 0.5 seconds delay time in the system using two-stage compensators. Moreover, the tracking performance of the system is improved. Figure 9 shows the step response for system with two-stage compensators with Ritz order of 1, 2, and 3. The results indicate that the maximum overshoot (objective function) can be improved by increasing Ritz order, while rise time and settling time meet their design specifications. Time domain performances of the FOPTD system with two-stage compensators shown in Table 5 confirms the results. Figure 10 shows the control signal which appears to be larger when increasing Ritz order. The larger magnitude of control can improve the performance of the closed-loop system. In addition, fig. 11 shows that system has low-sensitivity to disturbances in the low-frequency range.

Table 6. Time domain performance of the closed-loop FOPTD system.

Design	Performance Index		
	Rise time (s)	**Settling time (s)**	**Maximum overshoot (%)**
Specification	1.425	4.300	Objective Function
N=1	0.5104	3.676	27.66
N=2	0.5846	3.111	1.0513
N=3	0.6151	1.4057	0.4030

The second compensator can be obtained using the Eq. (9), (19), (20), (28), (29) and (30) as follows

$$C_2 - C_1 = \frac{(s+3)^N + x_1 s(s+3)^{N-1} + \cdots + x_N s}{(s+3)^N - \hat{G}((s+3)^N + x_1 s(s+3)^{N-1} + \cdots + x_N s)}.$$

The delay of system results in internal delays in the second compensator. Moreover, the complexity of C_2 depends on the Ritz approximation order. Increasing Ritz approximation order gives better performance.

4.3. Discussions

From the results, one of the interesting benefits of using two-stage compensators is that the design with multiple-objectives is expedient and effective. In contrast to typical PID controller, stability and time domain performance may be obtained, but complex tuning of PID parameters is required. In two-stage compensators design, tuning the controller is much simpler as we can choose any controller C_1 to ensure stability of the system (i.e., unity feedback for 2nd order system or PI controller for FOPTD system), and design other objectives with convex optimization in the 2nd controller. In addition, designing system with additional time constraints is possible if the specifications are convex.

5. Conclusions

This paper presents the method to design two-stage compensator such that the closed-loop system achieves its objectives in terms of stability, low sensitivity, and good time-domain performance. By applying Q-Parameterization and Ritz approximation, we transform the two-stage compensator design problem to convex optimization problem. In addition, the designed two-stage compensator makes the closed-loop system stable and obtains low steady-state error. For the time-domain performance, the optimal decision variables are obtained by solving convex optimization while other design criteria are still satisfied. Especially, the frequency response of the sensitivity function shows that the control system is not sensitive to disturbance. Hence, this research succeeds its objective that the designed two-stage compensator improves the performance of the control system. The ongoing research will consider the minimal realization to reduce the order of compensator.

Acknowledgement

This research is supported by Rachadapisek Sompote Fund, Chulalongkorn University for Intelligent Control Automation of Process Systems Research Unit. We are grateful to the Department of Electrical Engineering, Faculty of Engineering, Chulalongkorn University for research facility.

References

[1] M. Vidyasagar, *Control System Synthesis: A Factorization Approach Part I*. Morgan & Claypool Publishers, 2011.

[2] K. Mori, "Capability of two-stage compensator design," in *Proceedings of International Conference on Control and Automation*, Santiago, Chile, December 2011.

[3] K. Mori, "Two-stage compensator design - a factorization approach," in *Proceedings of 2008 SICE Annual Conference*, Chofu, Japan, September 2008, pp. 1310-1313.

[4] D. Youla, H. Jabr, and J. J. Bongiorno, "Modern Wiener-Hopf design of optimal controllers-Part II: The multivariable case," *IEEE Trans. Automatic Control*, vol. 21, no. 3, pp. 319-338, 1976.

[5] I. Mahtout, F. Navas, V Milanés, and F Nashashibi, "Advances in Youla-Kucera parameterization: A review," *Annual Reviews in Control*, vol. 49, pp. 81-94, 2020.

[6] Y. Choi and W. K. Chung, "Standard H_∞ state-space solution by Youla parameterization," *IFAC Proceedings*, vol. 30, no. 6, pp. 691-696, May 1997.

[7] L. Cui, "Youla parameterization based adaptive controller design for fault tolerant control," *Asian Journal of Control*, vol. 16 no. 1, pp. 292-295, 2014.

[8] S. Sabau and N. C. Martins, "Youla-like parameterizations subject to QI subspace constraints," *IEEE Transactions on Automatic Control*, vol. 59, no. pp. 1411-1422, June 2014.

[9] A. Linnemann, "Convergent Ritz approximations of the set of stabilizing controllers," *Systems & Control Letters*, vol. 36, no. 2, pp. 151-156, Feb. 1999.

[10] W. Reinelt, "Design of optimal control systems with bounded control signals," *Proceedings of European Control Conference*, Proto, Portugal, September 2001.

[11] L. Furieri, Y. Zheng, A. Papachristodoulou, and M. Kamgarpour, "An input-output parameterization of stabilizing controllers: Amidst Youla and system level synthesis," *IEEE Control Systems Letters*, vol. 3, no. 4, pp. 1014-1019, Oct. 2019.

[12] S. Ahmed and J. F. Whidborne, "Integration of H_∞ and time domain specification using the convex combination method," *IFAC Proceedings*, vol. 40, no. 7, pp. 135-140, 2007.

[13] N. Wongyimyong, T. Morikawa, K. Hashikura, T. Suzuki, D. Banjerdpongchai, and K. Yamada, "Design of two-stage compensators for simultaneous stabilization and low sensitivity," in *Proceedings of the First International Symposium of Instrumentation, Control, Artificial Intelligence and Robotics*, Bangkok, Thailand, January 2019.

[14] T. Morikawa, N. Wongyimyong, K. Hashikura, T. Suzuki, D. Banjerdpongchai, and K. Yamada, "Parameterization of simultaneously stabilizing controllers for two-stage compensator systems," *ICIC Express Letters*, vol. 13, no. 8, pp. 729-734, Aug. 2019.

[15] S. Boyd and C. Barratt, *Linear Controller Design: Limits of Performance*. New Jersey: Prentice Hall, 1991.

[16] W. Khaisongkram and D. Banjerdpongchai, "MATLAB based GUIs for linear controller design via convex optimization," *Computer Applications in Engineering Education*, vol. 11, no. 1, pp. 13-24, 2003.

[17] M. Grant and S. Boyd, *The CVX Users' Guide, Release 2.0*. Stanford University, July 2013.

[18] D. Adionel Guimaraes, G. H. Faria Floriano, and L. Silvestre Chaves, "A tutorial on the CVX system for modeling and solving convex optimization problems," *IEEE Latin America Transactions*, vol. 13, no. 5, pp. 1228-1257, May 2015.

[19] M. Bokasova, K. Vanekova, and J. Zavacka, "PI controller design for time delay systems," *IFAC Proceedings*, vol. 43, no. 2, pp. 349-354, 2010.

[20] M. Viteckova and A. Vitecek, "Controller tuning for FOPTD plants," in *Proceedings of the 13th International Carpathian Control Conference (ICCC)*, High Tatras, Slovakia, 2012, pp. 763-767.

[21] V. P. Mahajan, A. B. Patil, and T. D. Gangurde, "PID tuning methods for FOPTD system," in *Proceedings of 2019 2nd International Conference on Intelligent Computing, Instrumentation and Control Technologies*, Kannur, India, 2019, pp. 1152-1155.

DOI:10.4186/ej.2021.25.8.153

Tanathorn Supithak was born in Bangkok, Thailand in 1999. He received the B.S. degree (First class honors) in Electrical Engineering from Chulalongkorn University, Thailand, in 2021. His study was focused on three majors including control systems, electronics, and telecommunication. His research interest is control system design of two-stage compensator, convex optimization, control application to binary distillation column. He is awarded Japanese government scholarship to pursue postgraduate study at the Graduate School of Information Science and Technology, The University of Tokyo.

Nicha Wongyimyong was born in Bangkok, Thailand in 1997. She received the B.S. degree (First class honors) in Electrical Engineering from Chulalongkorn University, Thailand, in 2018. She had studied for two months as a research trainee at Graduate School of Science and Technology, Gunma University, Gunma, Japan in 2018. She was a coauthor of techincal papers presented at ICA-SYMP 2019 and ICIC Express Letters. Her research interests are control system design of two stage compensatores and convex optimization. She is currently working at Hitachi Elevator Co., Ltd., Thailand.

Kou Yamada was born in Akita, Japan, in 1964. He received B.S. and M.S. degrees from Yamagata University, Yamagata, Japan, in 1987 and 1989, respectively, and the Dr. Eng. degree from Osaka University, Osaka, Japan in 1997. From 1991 to 2000, he was with the Department of Electrical and Information Engineering, Yamagata University, Yamagata, Japan, as a research associate. From 2000 to 2008, he was an associate professor in the Department of Mechanical System Engineering, Gunma University, Gunma, Japan. Since 2008, he has been a professor in the Department of Mechanical System Engineering, Gunma University, Gunma, Japan. His research interests include robust control, repetitive control, process control and control theory for inverse systems and infinite-dimensional systems. Dr. Yamada received the 2005 Yokoyama Award in Science and Technology, the 2005 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON 2005) Best Paper Award, the Japanese Ergonomics Society Encouragement Award for Academic Paper in 2007, the 2008 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology International Conference (ECTI-CON 2008) Best Paper Award and Fourth International Conference on Innovative Computing, Information and Control Best Paper Award in 2009.

David Banjerdpongchai received his B.Eng. degree (First class honors) from Chulalongkorn University, M.S. and Ph.D. degrees from Stanford University, all in Electrical Engineering. Since 1990, he has been with the Department of Electrical Engineering, Chulalongkorn University. Currently, he is a professor of Electrical Engineering, the head of Control Systems Research Laboratory, and the head of Intelligent Control Automation of Process Systems Research Unit. He is a senior member of IEEE, a founding chair of IEEE Control Systems Society Thailand Chapter, and an executive committee member of ECTI Association. He serves as an associate editor of IJCAS and a section editor-in-chief of ASEAN Engineering Journal. His research interests are advanced process control, robust control systems, and energy management systems.