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Article

State Estimation Filtering using Recent Finite Measurements and Inputs for Active Suspension System with Temporary Uncertainties

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Abstract. In this paper, the finite memory structure(FMS) filter using most recent finite measured outputs and control inputs is applied for the state estimation filtering of automotive suspension systems to verify intrinsic robustness of FMS filter. Firstly, the single-corner model for the automotive suspension system and its state-space model are described. Secondly, FMS as well as infinite memory structure(IMS) filters are briefly introduced and represented by the summation form. Thirdly, a couple of temporary uncertainties, model uncertainty and unknown input, are discussed. Finally, extensive computer simulations are performed for both nominal system and temporarily uncertain system. It is shown that the FMS filter can be better than the IMS filter for both temporary uncertainties. In addition, the FMS filter can be shown to be comparable to the IMS filter after the effects of a couple of temporary uncertainties have completely disappeared.

Keywords: Infinite memory structure filter, finite memory structure filter, automotive suspension system, temporary uncertain system, nominal system.

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1. Introduction

In recent years, automotive industry has shown a trend to replace electromechanical components by intelligent and autonomous mechatronic systems because of the increasing mobility requirements. Hence, one of the critical components in the present of motor vehicles, an active suspension system has attracted many researchers in the past few decades [1]-[6]. These researches have paid considerable attention to the issues on how to reduce acceleration of sprung mass continuously as well as to minimize suspension deflection, which leads guarantee of the stability of the suspension systems and improvement of the required suspension performances such as ride comfort, road handling and suspension deflection, vehicle maneuverability.

In the automotive suspension system, the state-space model has been used for control and estimation problems [7]-[12]. In real situations, the automotive suspension system can contain system and measurement noises which can cause the system error. Hence, for the state-space model of the automotive suspension system with noises, a state estimation filter should be applied to estimate the proper state of the controller in order that the suspension performance is to be improved. Generally, the state estimation filter can be classified by the infinite memory structure(IMS) filter and the finite memory structure(FMS) filter according to the measurement processing manner. The IMS filter, such as the well-known Kalman filter [13]-[16], has been successfully applied for the automotive suspension system [9]-[12]. In contrast to the IMS filter, the FMS filter was developed using only finite measured outputs on the most recent window [17]-[20]. The FMS filter has been applied successfully for various engineering problems [21]-[26].

Even if the automotive suspension system is accurately represented in state-space model on a long time scale, unpredictable changes such as frequency, phase, and speed jumps can occur. This effect is called temporary uncertainties because it generally occurs in the short term [17]-[20]. As representative temporary uncertainties, there are a model uncertainty, an unknown input, and incomplete measurement information, etc. The state estimation filter for dynamic systems should be robust to diminish the effects of these temporary uncertainties.

Due to its memory structure and measurement processing manner, the FMS filter has been known inherently to be bounded input/bounded output stable and robust against temporary uncertainties, which means that the FMS filter has an intrinsic robustness property. In this paper, the FMS filter using most recent finite measured outputs and control inputs is applied for the state estimation filtering of automotive suspension systems under temporary uncertainties to verify intrinsic robustness of FMS filter. Firstly, the single-corner model for the automotive suspension system and its state-space model are described. Secondly, both FMS and IMS filters are briefly introduced and represented by the summation form. Thirdly, a model uncertainty and an unknown input are discussed as representative temporary uncertainties. Finally, to verify intrinsic robustness of FMS filter, extensive computer simulations are performed for both nominal system and temporarily uncertain system. It is shown that the FMS filter can be better than the IMS filter for a couple of temporary uncertainties, which means that people sitting in the vehicle with FMS filter can be more comfortable due to the smaller estimation error magnitude and shorter estimation error convergence time than the vehicle with IMS filter. In addition, the FMS filter can be shown to be comparable to the IMS filter after the effects of a couple of temporary uncertainties have completely disappeared.

This paper has the following structure. In Section 2, an automotive suspension system and its state-space model are described. In Section 3, FMS and IMS estimation filters with summation forms are introduced briefly. In Section 4, a couple temporary uncertainties are discussed. In Section 5, extensive computer simulations are performed. Then, concluding remarks are given in Section 6.

2. Automotive Suspension System and Its State-Space Model

The automotive suspension system allows wheel movement independent of the automotive body, which can isolate automotive body from road disturbances such as bumps and potholes. Both spring and damper try to remove the effects of road disturbances on the ride as well as stability of the automotive. The spring manipulates the frequency of road disturbances and tries to bring them into a more manageable band. The damper dissipates the energy of the dynamic load coming through road disturbances. Hence, designing an automotive suspension system is a very interesting and challenging in control problems. When the automotive suspension system is designed, the single-corner model for one of the four wheels, called the quarter car model, is used to simplify the problem to a 1-D multiple spring-damper system. The single-corner model for the active suspension system including an actuator is able to generate the control force to control the motion of the automotive body. In this paper, a bus suspension system is considered and its diagram is shown in Figure 1 [5].

From Figure 1 and Newton's law, the following dynamic equations can be obtained:

$$\begin{aligned} M_1\ddot{X}_1 &= -b_1(\dot{X}_1-\dot{X}_2)-K_1(X_1-X_2)+u, \\ M_2\ddot{X}_2 &= b_1(\dot{X}_1-\dot{X}_2)-K_1(X_1-X_2) \\ &\quad +b_2(\dot{p}-\dot{X}_2)+K_2(p-X_2)-u, \end{aligned} \tag{1}$$

where the automotive suspension system consists of several variables and parameters as shown in Table 1.

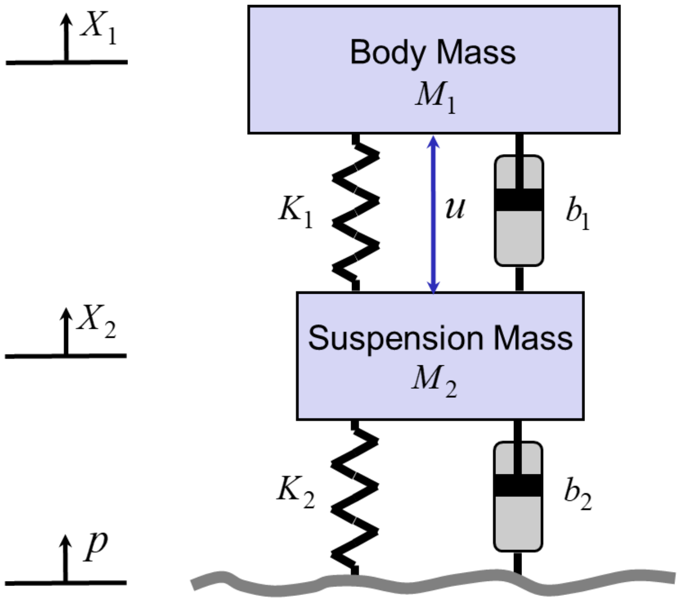

Fig. 1. Model for automotive suspension system.

Table 1. Variables and parameters of automotive suspension system

Variables	
$X_1(t)$	body displacement
$X_2(t)$	suspension displacement
$u(t)$	desired input force by the cylinder
$p(t)$	road disturbance such as bump and pothole
Parameters	
M_1	body mass [kg]
M_2	suspension mass [kg]
K_1	spring constant of suspension system [N/m]
K_2	spring constant of wheel and tire [N/m]
b_1	damping constant of suspension system [$N.s/m$]
b_2	damping constant of wheel and tire [$N.s/m$]

The dynamic equation (1) of the automotive suspension system can be represented in the state-space model. The state-space approach has been a general method for modeling, analyzing and designing a wide range of control systems in time-domain and is especially suitable for digital computation techniques. In this paper, the state-space realization is also required for the automotive suspension system to apply the state estimation filtering.

A continuous-time state-space model of the automotive suspension system can be represented by

$$\begin{aligned} \dot{x}(t) &= A_c x(t) + B_c \left[\begin{array}{c} u(t) \\ p(t) \end{array} \right], \\ y(t) &= C_c x(t), \end{aligned} \tag{2}$$

where variables and parameters are expressed by

$$\begin{aligned} x(t) &= \left[\begin{array}{c} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{array} \right] = \left[\begin{array}{c} X_1(t) \\ \dot{X}_1(t) \\ Y_1(t) \\ \dot{Y}_1(t) \end{array} \right], \\ y(t) &= Y_1(t) = X_1(t) - X_2(t), \end{aligned}$$

$$A_c = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ \frac{-b_1 b_2}{M_1 M_2} & 0 & \left[\frac{b_1}{M_1} \left(\frac{b_1}{M_1} + \frac{b_1}{M_2} + \frac{b_2}{M_2} \right) - \frac{K_1}{M_1} \right] & -\frac{b_1}{M_1} \\ \frac{b_2}{M_2} & 0 & -\left(\frac{b_1}{M_1} + \frac{b_1}{M_2} + \frac{b_2}{M_2} \right) & 1 \\ \frac{K_2}{M_2} & 0 & -\left(\frac{K_1}{M_1} + \frac{K_1}{M_2} + \frac{K_2}{M_2} \right) & 0 \end{array} \right],$$

$$
\begin{aligned}
B_c &= \begin{bmatrix} 0 & 0 \\ \frac{1}{M_1} & \frac{b_1 b_2}{M_1 M_2} \\ 0 & -\frac{b_2}{M_2} \\ \left(\frac{1}{M_1}+\frac{1}{M_2}\right) & \frac{K_2}{M_2} \end{bmatrix}, \\
C_c &= \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}.
\end{aligned}
$$

The body velocity $\dot{X}_1(t)$ and the damper velocity $\dot{Y}_1(t) = \dot{X}_1(t) - \dot{X}_2(t)$ are important variables in the automotive suspension control design. A feedback controller is designed in order that the output $X_1(t) - X_2(t)$ for the relative displacement between body and suspension should meet design specifications such as an overshoot and a settling time. Thus, this paper focuses on estimation filtering of two velocities and the relative displacement between body and suspension using measured outputs from the wheel displacement sensor.

3. Two Kinds of State Estimation Filters

The continuous-time state-space model (2) is discretized with the sample time. Since the occurrence of road disturbances such as bump and pothole is unpredictable, the road disturbance term p_i can be considered as temporary uncertainty and thus be ignored in the nominal discrete-time state-space model. In addition, there can be system and measurement noises in real situations. Thus, the discretized system for the automotive suspension system can be extended by the following discrete-time state-space model

$$
\begin{aligned}
x_{i+1} &= Ax_i + Bu_i + Gw_i, \\
y_i &= Cx_i + v_i,
\end{aligned} \tag{3}
$$

where

$$
\begin{aligned}
A &= e^{A_c T}, \\
\begin{bmatrix} B & E \end{bmatrix} &= \left(\int_0^T e^{A_c \varepsilon} d\varepsilon\right) B_c \\
&= \left(e^{A_c T} - I\right) A_c^{-1} B_c, \\
C &= C_c,
\end{aligned} \tag{4}
$$

and the matrix E related to the road disturbance will be used in the next section and the matrix G related to the system noise is assumed to be same as E in this paper. The initial state $\hat{x}_{i_0}$ is a random variable with a mean $\bar{x}_{i_0}$ and a covariance Σ_{i_0}. The w_i is the system noise and v_i is the measurement noise in the wheel displacement sensor. These noises are zero-mean white Gaussian whose covariances Q and R are assumed to be positive definite matrix. The desired input force u_i by the cylinder is treated as a control input. Since the road disturbance can be considered as temporary uncertainty, the term p_i is excluded. The state variable x_i and output variable y_i are specified as shown in Table 2.

As shown in (3), the automotive suspension system contains system and measurement noises which can cause the system error. Hence, the state estimation filtering has been applied to estimate state variables of the automotive suspension system with noises using measured outputs from the wheel displacement sensor measuring relative displacement between body and suspension. Generally, the state estimation filter can be classified by the infinite memory structure(IMS) filter and the finite memory structure(FMS) filter according to the measurement processing manner as follows.

3.1. Infinite Memory Structure (IMS) Filter

The IMS filter, such as the well-known Kalman filtering [13]-[16], has been successfully applied for the automotive suspension system [9]-[12]. The IMS filter provides an optimal state estimate $\hat{x}_i^{ims}$ for the system state x_i using all past measured outputs. The IMS filtered estimate $\hat{x}_i^{ims}$ can be represented by the summation form with the initial condition $\hat{x}_{i_0}^{ims} = \bar{x}_{i_0}$ as follows:

$$
\begin{aligned}
\hat{x}_i^{ims} &= \Phi_i \hat{x}_{i_0}^{ims} + \sum_{j=0}^{i-1} \Phi_{i-j} \Sigma_j C^T R^{-1} y_j \\
&\quad + \sum_{j=0}^{i-1} \Phi_{i-j} B u_j \\
&= \Phi_i \bar{x}_{i_0} + \sum_{j=0}^{i-1} \Phi_{i-j} \Sigma_j C^T R^{-1} y_j \\
&\quad + \sum_{j=0}^{i-1} \Phi_{i-j} B u_j,
\end{aligned} \tag{5}
$$

where the transition matrix Φ_j is given by

$$
\begin{gathered}
\Phi_{j+1} = \Phi_j A \big[I + \Sigma_{i-j-1} C^T R^{-1} C\big]^{-1}, \\
\Phi_0 = I, \\
i_0 \le j \le i-1,
\end{gathered}
$$

and the error covariance Σ_i of the estimate $\hat{x}_i^{ims}$ is given by

$$
\Sigma_{i+1} = A\big[I + \Sigma_i C^T R^{-1} C\big]^{-1} \Sigma_i A^T + GQG^T,
$$

with the initial value Σ_{i_0} which is the covariance of $\hat{x}_{i_0}^{ims}$.

3.2. Finite Memory Structure (FMS) Filter

As an alternative to the IMS filter, the FMS filter has been developed [17]-[20]. The FMS filter provides an optimal state estimate $\hat{x}_i^{fms}$ for the system state x_i using only the most recent finite measured outputs and control inputs on the window $[i - M, i]$. The window initial time $i - M$ will be denoted by i_M hereafter for simplicity. The FMS

Table 2. State and output variables for automotive suspension system

State variables	
x_1	body displacement
x_2	body velocity
x_3	relative displacement between body and suspension
x_4	relative velocity between body and suspension (i.e. damper velocity)
Output variables	
$y = x_3$	signals from wheel displacement sensors measuring relative displacement between body and suspension

filtered estimate $\hat{x}_i^{fms}$ can be represented by the summation form as well as the matrix form with the window initial condition $\hat{x}_{i_M} = \big(\Gamma^T\Pi^{-1}\Gamma\big)^{-1}\Gamma^T\Pi^{-1}Y_i$ as follows:

$$\begin{aligned}
\hat{x}_i^{fms} &= \Phi_M \hat{x}_{i_M} + \sum_{j=0}^{M-1}\Phi_{M-j}\Sigma_j C^T R^{-1} y_{i_M+j} \\
&\quad + \sum_{j=0}^{M-1}\Phi_{M-j} B u_{i_M+j} \\
&= \Phi_M\Big(\Gamma^T\Pi^{-1}\Gamma\Big)^{-1}\Gamma^T\Pi^{-1}Y_i \\
&\quad + \Big[\Phi_M\Sigma_0 \;\; \Phi_{M-1}\Sigma_1 \;\; \cdots \;\; \Phi_1\Sigma_{M-1}\Big]C^T R^{-1} Y_i \\
&\quad + \Big[\Phi_M \;\; \Phi_{M-1} \;\; \cdots \;\; \Phi_1\Big] B U_i,
\end{aligned} \tag{6}$$

where the transition matrix Φ_j is given by

$$\begin{aligned}
\Phi_{j+1} &= \Phi_j A\big[I + \Sigma_{M-j-1}C^T R^{-1} C\big]^{-1}, \\
\Phi_0 &= I, \;\; 0 \le j \le M-1,
\end{aligned}$$

and the error covariance Σ_j of the estimate $\hat{x}_i^{fms}$ is given by

$$\begin{aligned}
\Sigma_{j+1} &= A(I + \Sigma_j C^T R^{-1} C)^{-1}\Sigma_j A^T + GQG^T, \\
\Sigma_0 &= \Big(\Gamma^T\Pi^{-1}\Gamma\Big)^{-1}.
\end{aligned}$$

The most recent finite measured outputs Y_i and control inputs U_i are as follows:

$$Y_i \triangleq \begin{bmatrix} y_{i_M} \\ y_{i_M+1} \\ y_{i_M+2} \\ \vdots \\ y_{i-1} \end{bmatrix}, \; U_i \triangleq \begin{bmatrix} u_{i_M} \\ u_{i_M+1} \\ u_{i_M+2} \\ \vdots \\ u_{i-1} \end{bmatrix},$$

and matrices Γ, Λ, and Π are as follows:

$$\begin{aligned}
\Gamma &\triangleq \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{M-1} \end{bmatrix}, \\
\Lambda &\triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ CG & 0 & \cdots & 0 & 0 \\ CAG & CG & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{M-2}G & CA^{M-3}G & \cdots & CG & 0 \end{bmatrix}, \\
\Pi &\triangleq \Lambda\Bigg[\mathrm{diag}(\overbrace{Q\,Q\,\cdots\,Q}^{M})\Bigg]\Lambda^T + \Bigg[\mathrm{diag}(\overbrace{R\,R\,\cdots\,R}^{M})\Bigg].
\end{aligned}$$

Gain matrices $\big(\Gamma^T\Pi^{-1}\Gamma\big)^{-1}\Gamma^T\Pi^{-1}$ and $\big[\Phi_M\Sigma_0 \;\; \Phi_{M-1}\Sigma_1 \;\; \cdots \;\; \Phi_1\Sigma_{M-1}\big]C^T R^{-1}$ for finite measured outputs Y_i and $\big[\Phi_M \;\; \Phi_{M-1} \;\; \cdots \;\; \Phi_1\big]B$ for finite control inputs U_i in (6) requires only once off-line computation on the interval $[0, M]$ and then is used for all windows, which means that the FMS filter has the time-invariance property.

4. Discussion on Temporary Uncertainties

Even if the automotive suspension system is accurately represented in state-space model on a long time scale, unpredictable changes such as frequency, phase, and speed jumps can occur. This effect is called temporary uncertainties because it generally occurs in the short term [17]-[20]. There can be a model uncertainty, an unknown input, and incomplete measurement information, etc., as representative temporary uncertainties. In this paper, the model uncertainty and the unknown input are considered.

Firstly, the model uncertainty is considered. The state-space approach is commonly used when real physical systems and processes can be approximated with a reasonable number of states. The approximation implies model uncertainty that may cause an estimator to be biased and/or diverge. That is, due to concerns for model misspecification, there can be model uncertainty. The automotive suspension system with the model uncertainty can be represented by

$$\begin{aligned} x_{i+1} &= (A + \Delta A_i)x_i + Bu_i + Gw_i, \\ y_i &= (C + \Delta C_i)x_i + v_i. \end{aligned} \tag{7}$$

Although IMS and FMS filters (5) and (6) are designed by the nominal discrete-time state-space model (3) for automotive suspension system, actual measured outputs for the estimation filtering are obtained from the system with the model uncertainty (7).

Secondly, the unknown input is considered. The unknown input has been used in many areas such as fault detection and isolation for various systems and maneuver detection and tracking of moving targets. In the automotive suspension system, road disturbances such as bump and pothole can be treated as the unknown input. As mentioned before, the road disturbance term p_i is excluded in the nominal discrete-time state-space model (3). However, actual measured outputs from the wheel displacement sensor contain the road disturbance. In other words, although IMS and FMS filters (5) and (6) are designed by the nominal discrete-time state-space model (3), actual measured outputs are obtained from the following state-space model with the road disturbance term

$$\begin{aligned} x_{i+1} &= Ax_i + Bu_i + Ep_i + Gw_i, \\ y_i &= Cx_i + v_i, \end{aligned} \tag{8}$$

where the matrix E related to the road disturbance can be obtained from (4)

The state estimation filter for the automotive suspension system should be robust to diminish the effects of these temporary uncertainties. The IMS filter such as the Kalman filter has been a standard choice and a beautiful reference for the state estimation and thus applied successfully for diverse engineering problems. However, due to its measurement processing manner that utilizes all past measured outputs accomplished by equal weighting, the IMS filter tends to accumulate during its implementation. Therefore, it is known that the IMS filter may show poor performance and even divergence phenomena for temporary uncertainties. In contrast to the IMS filter, the FMS filter using only finite measured outputs and control inputs on the most recent window has been known inherently to be bounded input/bounded output stable and more robust against temporary uncertainties. This means that the FMS filter has an intrinsic robustness property. To verify intrinsic robustness of FMS filter, the FMS filter is applied for the state estimation filtering of automotive suspension systems under a couple of temporary uncertainties.

5. Verification for Intrinsic Robustness of FMS Filter

In this section, to verify intrinsic robustness of FMS filter, a bus suspension system is considered for computer simulations with physical parameters as shown in Table 3 [5]. Both FMS and IMS filters are applied for the estimation filtering of the bus suspension system and then compared.

System matrices for the discrete-time state-space model of the automotive suspension system can be obtained by discretization with physical parameters in Table 3 as follows:

$$\begin{aligned} A &= \begin{bmatrix} 0.8628 & 0.0958 & -0.0044 & -0.0013 \\ -2.8740 & 0.8628 & 0.1998 & -0.0178 \\ 0.7829 & 0.0864 & -0.0648 & -0.0021 \\ -9.7230 & 0.4498 & 1.9570 & -0.1183 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.0000018 \\ 0.0000337 \\ 0.0000036 \\ 0.0001143 \end{bmatrix}, \\ G &= \begin{bmatrix} 0.1372 \\ 2.8740 \\ -0.7829 \\ 9.7230 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}. \end{aligned} \tag{9}$$

The system noise covariance and the measurement noise covariance are taken by $Q = 0.01^2$ and $R = 0.02^2$, respectively. To make a clearer comparison of estimation performances, simulations of 20 runs are performed using different noises, and each single simulation run lasts 500 samples.

For each simulations, estimation errors are computed for three kinds of state variables, the body velocity(2nd state), the damper velocity(4th state), and the relative displacement between body and suspension(3rd state). These state variables are important variables for the automotive suspension control design and for the measuring passengers' ride comport.

5.1. Simulation Results for Nominal System

For the nominal discrete-time state-space model (3) with (9) where there is no temporary uncertainty, two filters are compared by root mean square(RMS) estimation error for simulations of 20 runs. As shown in Figure 2, the FMS filter can be comparable to the IMS filter. That is, both filters produce negligible errors for the nominal system.

5.2. Simulation Results for Temporary Uncertainties

Firstly, to consider the temporary uncertainty for the automotive suspension system, a model uncertainty ΔA_i and ΔC_i in (7) is considered as follows:

$$\begin{aligned} \Delta A_i &= \delta_i \cdot I_{4\times 4}, \;\; \Delta C_i \;=\; 0.1\delta_i \cdot C, \\ \delta_i &= \begin{cases} 0.05 & \text{if } 150 \le i \le 200, \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{10}$$

Table 3. Physical parameters of bus suspension system for simulations

Parameters	Values
M_1	2,500 [*kg*]
M_2	320 [*kg*]
K_1	80,000 [*N/m*]
K_2	500,000 [*N/m*]
b_1	350 [*N.s/m*]
b_2	15,020 [*N.s/m*]

Secondly, a road disturbance p_i in (8) is considered and simulated by two kinds of 20 *cm* step disturbances, bump and pothole, as follows:

$$p_i = \begin{cases} 0.2 & \text{if } 150 \leq i \leq 200, \\ -0.2 & \text{if } 350 \leq i \leq 400, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

where $E = G$. These two kinds of step type road disturbances could represent the bus coming out of bump and pothole.

Although both IMS and FMS filters are designed by the nominal discrete-time state-space model (9) for the automotive suspension system, actual measured outputs for these two filters are obtained from the system with temporary uncertainties (10) and (11). Upper plots of Figures 3-8 show RMS estimation errors of the body velocity, the damper velocity, and the relative displacement between body and suspension for 20 simulations for a couple of temporary uncertainties. In addition, lower plots of Figures 3-8 also show estimation errors for one of 20 simulations.

As shown in simulation results, the FMS filter can be better than the IMS filter in terms of error magnitude and error convergence. The estimation error of FMS filters is smaller than that of the IMS filter on the interval where model uncertainty or road disturbance exist temporarily. In addition, the convergence of estimation error is faster than that of the IMS filter after model uncertainty or road disturbance disappears. Meanwhile, the FMS filter can be comparable to the IMS filter after the effects of model uncertainty and road disturbance have completely disappeared.

This means that people sitting in the bus with FMS filter can be more comfortable due to the smaller error magnitude and shorter error convergence time than the bus with IMS filter. Therefore, the FMS filter can be more robust than the IMS filter when applied to the automotive suspension system with model uncertainty or road disturbance, although the FMS filter is designed with no consideration of robustness. This observation means that the FMS filter has an intrinsic robustness property.

6. Conclusions

This paper has applied the FMS filter for the estimation filtering of the automotive suspension systems to verify intrinsic robustness of FMS filter. Firstly, the single-corner model for the automotive suspension system and its state-space model have been described. Secondly, both FMS and IMS filters have been briefly introduced and compared. Thirdly, a couple of temporary uncertainties such as model uncertainty and unknown input has been discussed. Finally, extensive computer simulations have been performed for both nominal system and temporarily uncertain system. It has been shown that the FMS filter can be better than the IMS filter for a couple of temporary uncertainties, which means that people sitting in the bus with FMS filter can be more comfortable due to the smaller overshoot and shorter settling time than the bus with IMS filter. In addition, the FMS filter has shown to be comparable to the IMS filter after the effects of model uncertainty and road disturbance have completely disappeared.

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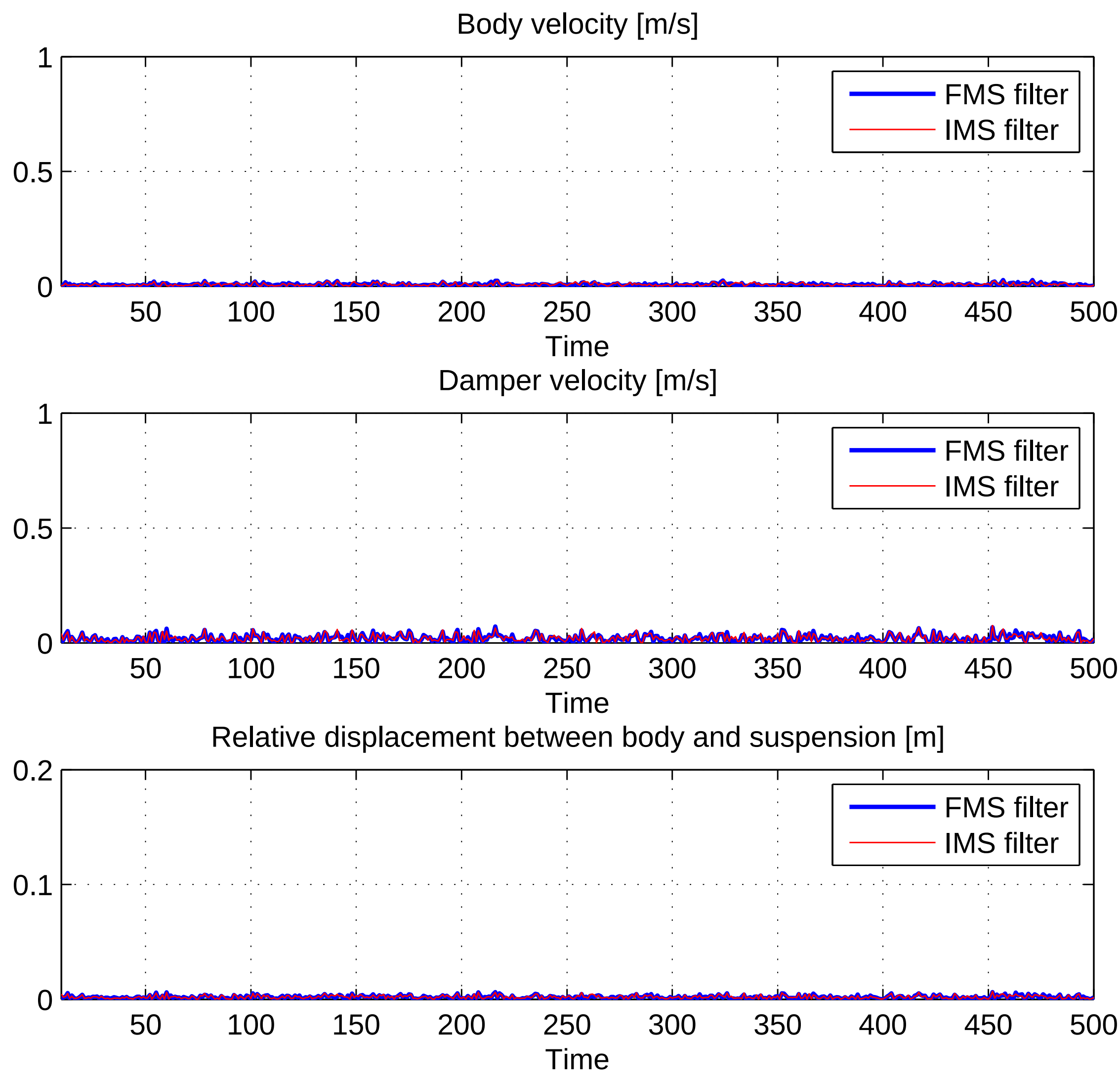

Fig. 2. RMS estimation errors of three kinds of state variables for nominal system.

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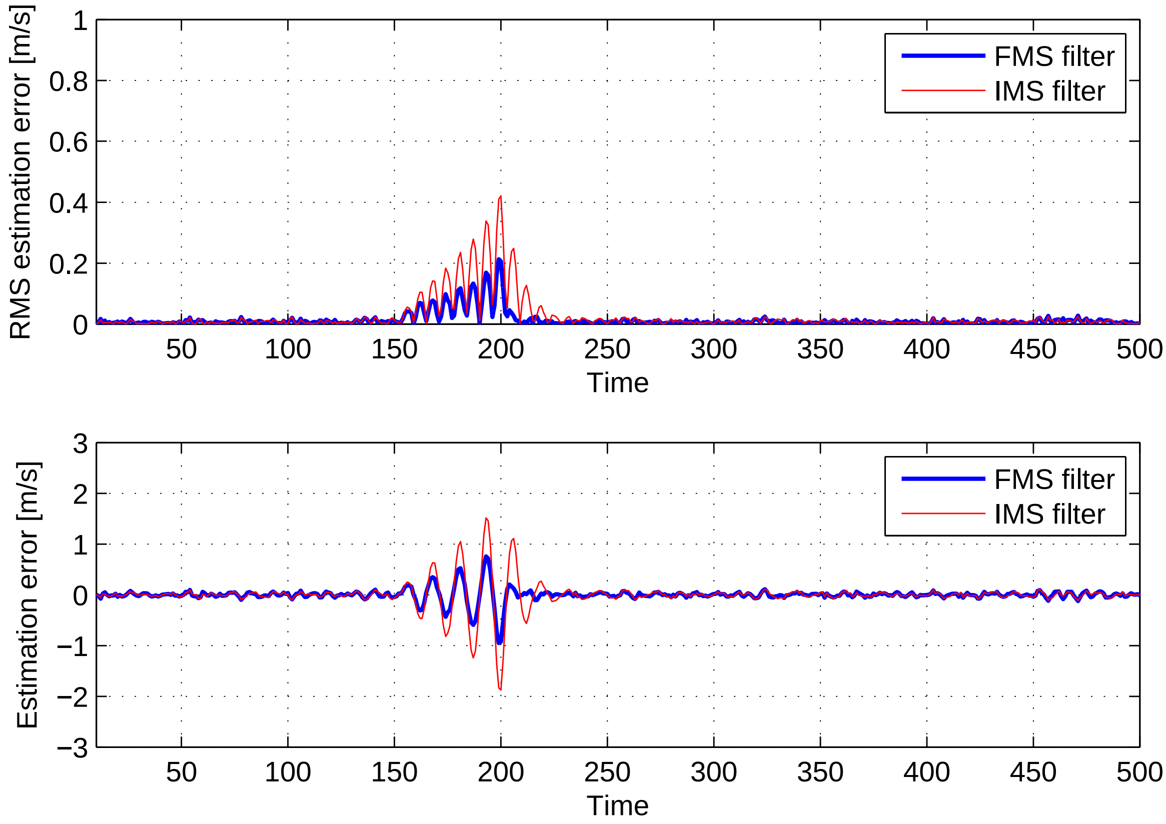

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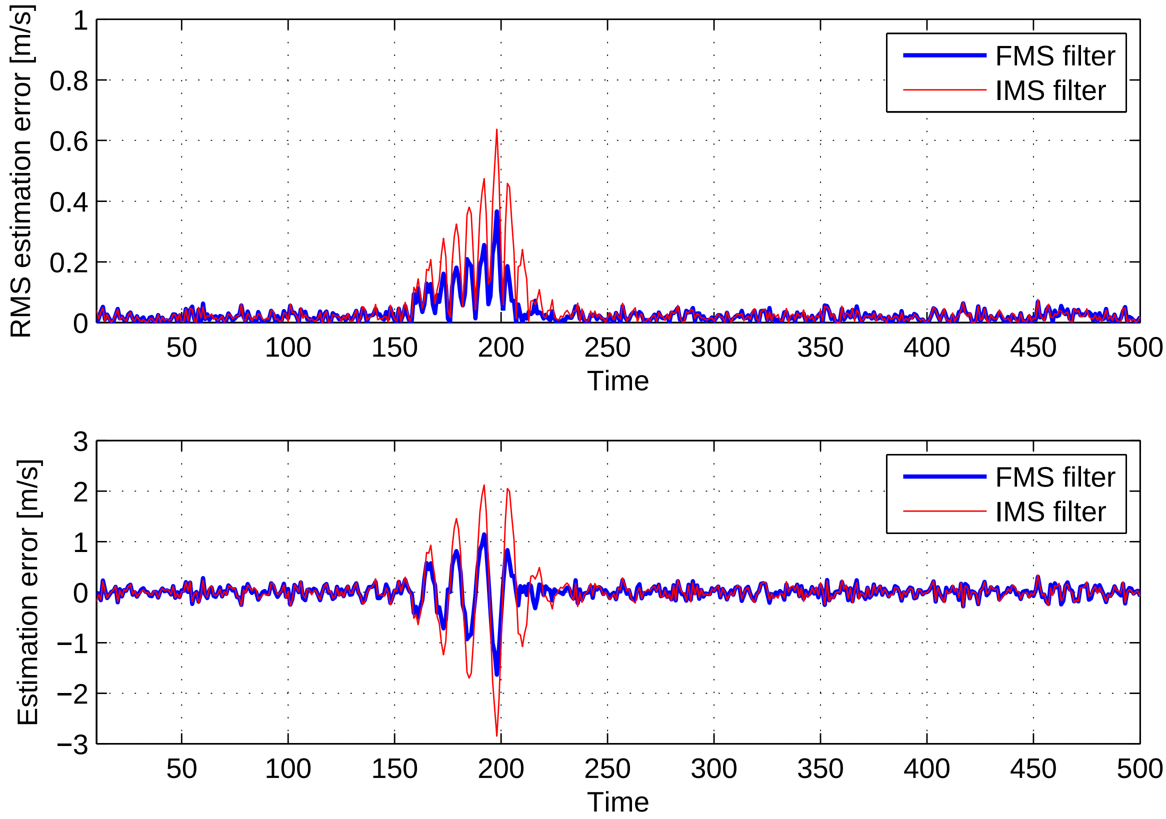

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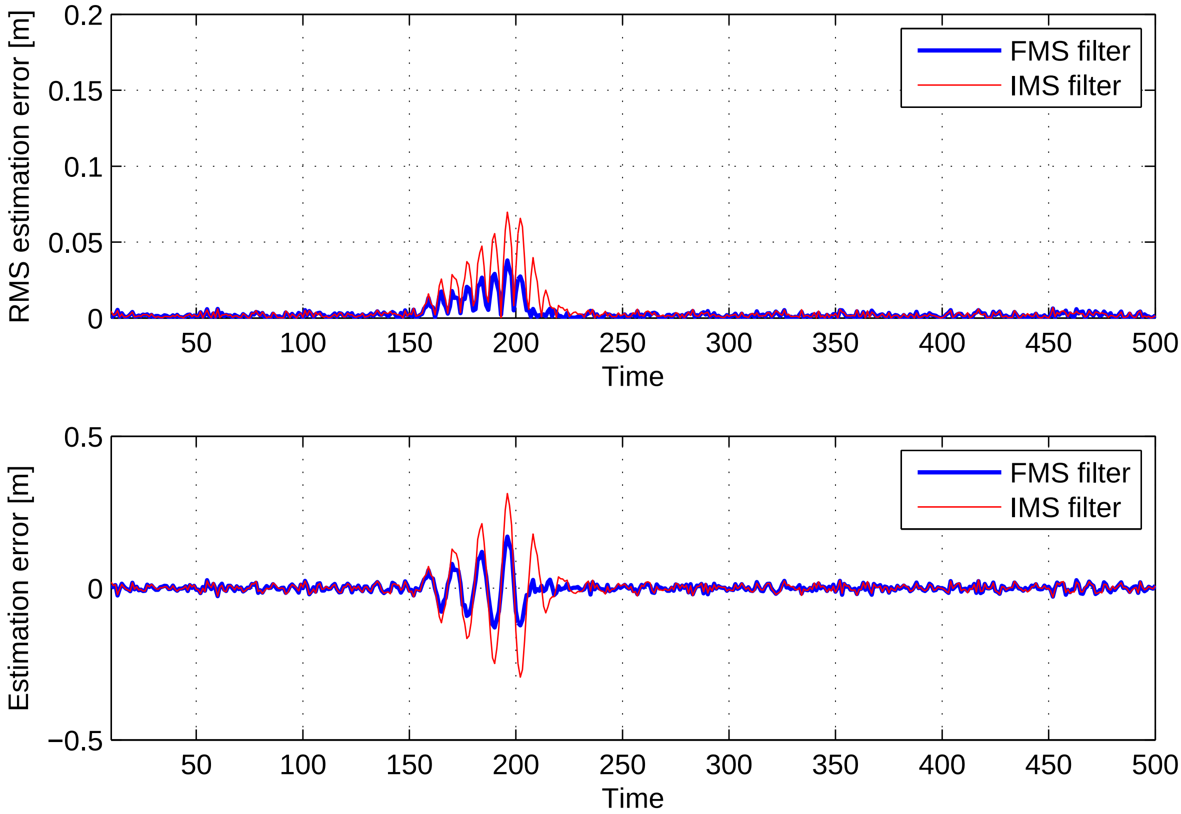

Fig. 5. Results of relative displacement between body and suspension for model uncertainty.

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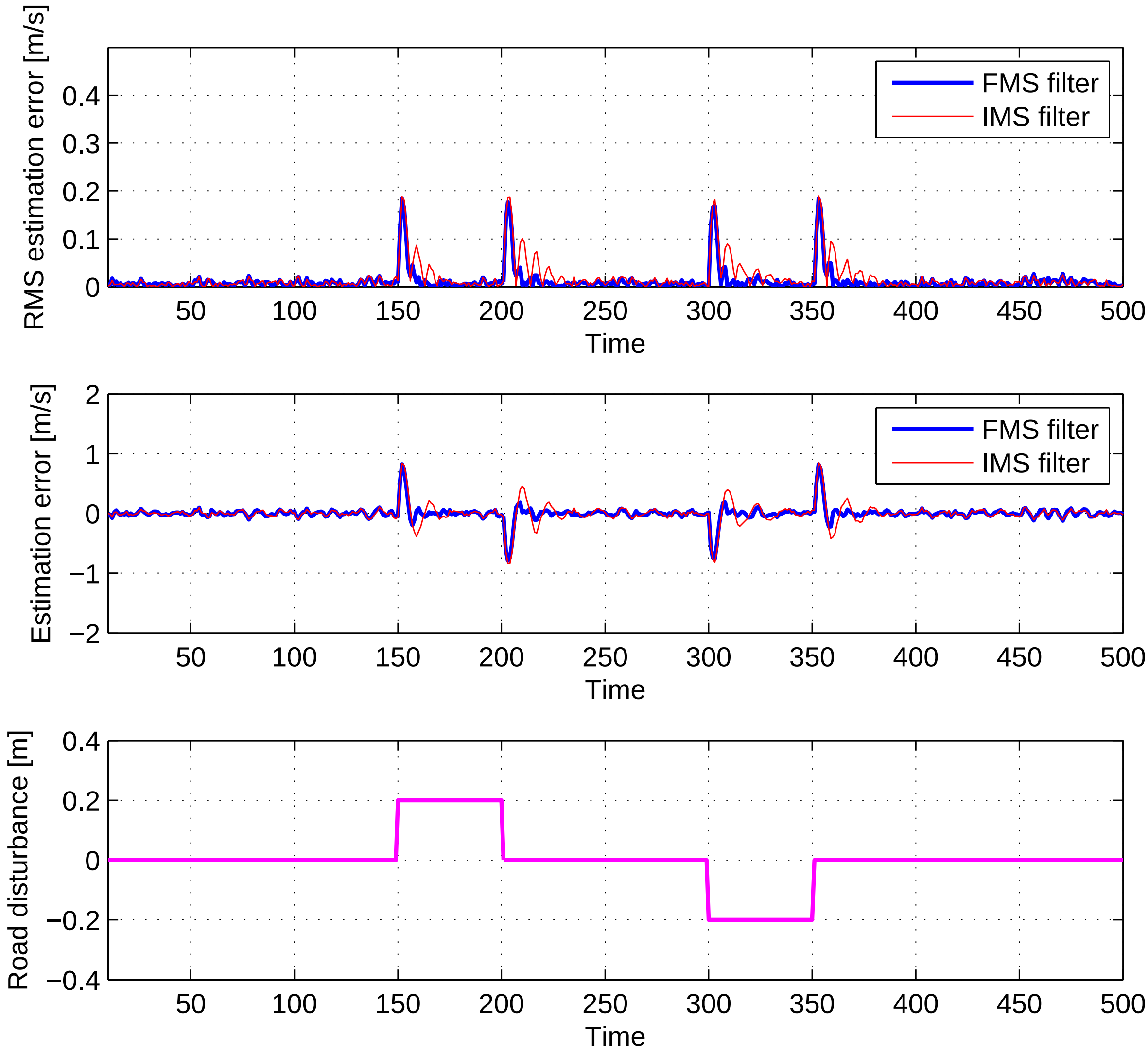

Fig. 6. Results of body velocity for road disturbance.

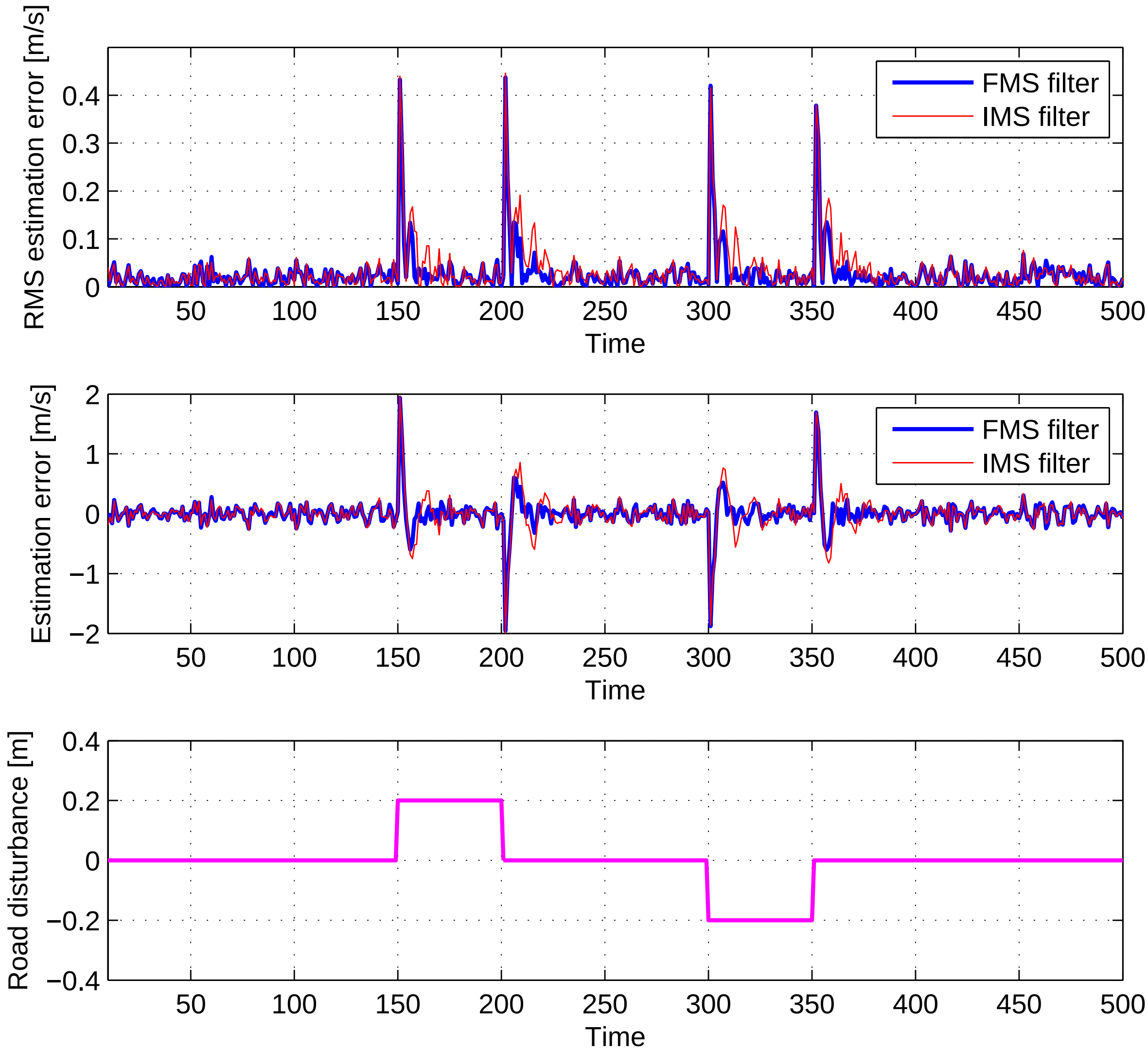

Fig. 7. Results of damper velocity for road disturbance.

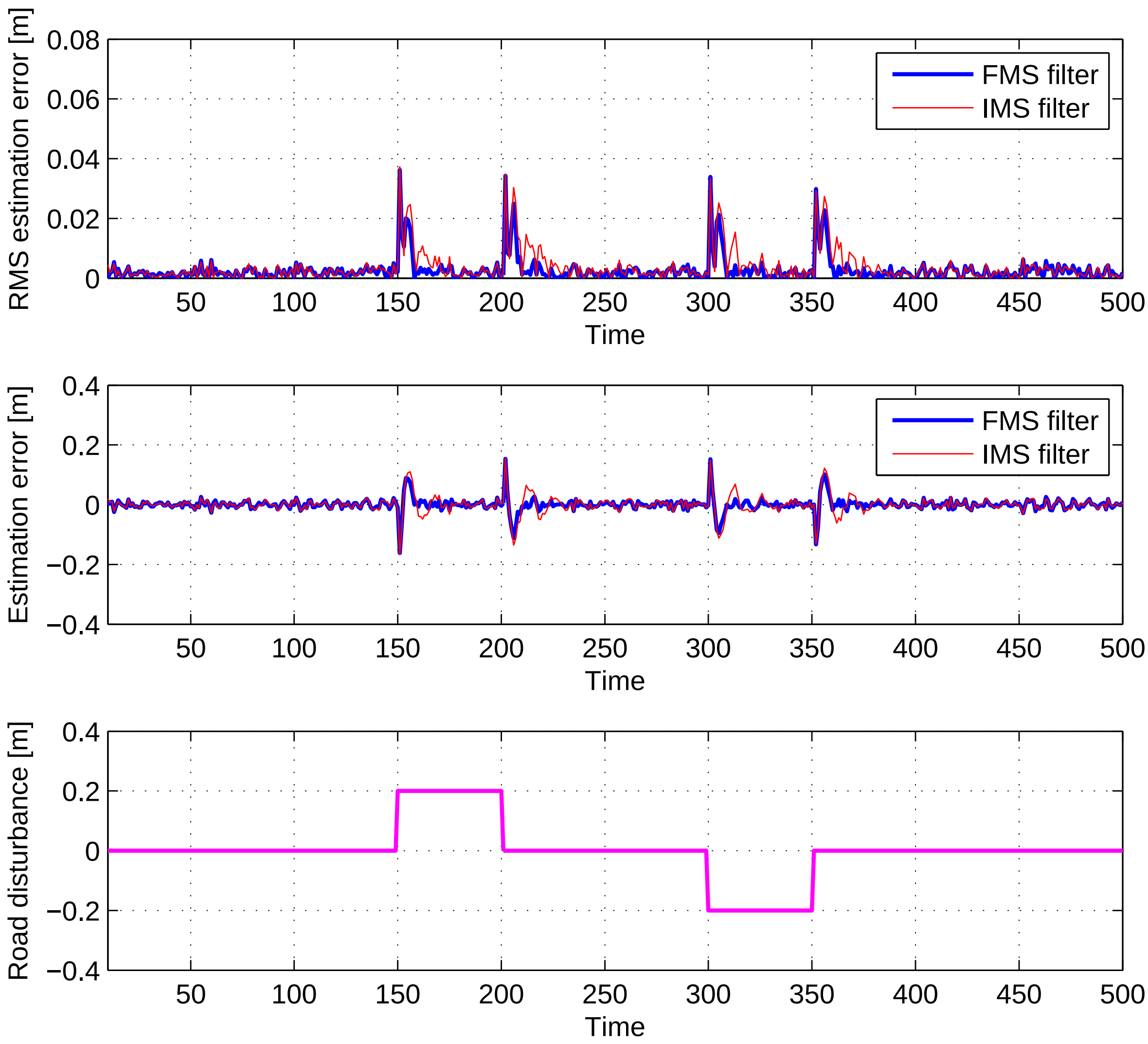

Fig. 8. Results of relative displacement between body and suspension for road disturbance.

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