Engineering Journal
issn: 0125-8281

Article

Consensus Synthesis of Robust Cooperative Control for Homogeneous Leader-Follower Multi-Agent Systems Subject to Parametric Uncertainty

Tuynh Van Pham[1,a], **Dinh Hoa Nguyen**[2,b], **and David Banjerdpongchai**[1,c,*]

1 Research Unit on Intelligent Control Automation of Process Systems, Department of Electrical Engineering, Faculty of Engineering, Chulalongkorn University, 254 Phayathai Road, Pathumwan, Bangkok 10330, Thailand
2 International Institute for Carbon-Neutral Energy Research, and Institute of Mathematics for Industry, Kyushu University, Fukuoka 819-0395, Japan
E-mail: [a]tuynhvanpham@gmail.com, [b]hoa.nd@i2cner.kyushu-u.ac.jp, [c]bdavid@chula.ac.th (Corresponding author)

Abstract. This paper presents a design of robust consensus for homogeneous leader-follower multi-agent systems (MAS). Each agent of MAS is described by a linear time-invariant dynamic model subject to parametric uncertainty. The agents are interconnected through a static interconnection matrix over an undirected graph to cooperate and share information with their neighbours. The consensus design of MAS can be transformed to stability analysis by using decomposition technique. We apply Lyapunov theorem to derive the sufficient condition to ensure the consensus of all independent subsystems. In addition, we design a robust distributed state feedback gain based on linear quadratic regulator (LQR) setting. Controller gain is computed via solving a linear matrix inequality. As a result, we provide a robust design procedure of a cooperative LQR control to achieve consensus objective and maximize the admissible bound of the uncertainty. Finally, we give numerical examples to illustrate the effectiveness of the proposed consensus design. The results show that the response for MAS in presence of uncertainty using robust consensus design follows the response of the leader and is better than that of the existing nominal consensus design.

Keywords: Cooperative LQR, multi-agent systems, parametric uncertainty, robust consensus control.

ENGINEERING JOURNAL Volume 24 Issue 3
Received 4 December 2019
Accepted 15 April 2020
Published 31 May 2020
Online at http://www.engj.org/
DOI:10.4186/ej.2020.24.3.169

This article is based on the presentation at The 2019 First International Symposium on Instrumentation, Control, Artificial Intelligence, and Robotics (ICA-SYMP 2019) in Bangkok, Thailand, 16 - 18 January 2019.

1. Introduction

Hierarchical control in large-scale network dynamical systems has been extensively investigated for a decade. The main reason is due to its wide application in real-world complex systems such as transportation, power grids, and social network. The fundamental hierarchical control is to attain global objectives by designing local controllers and cooperating among subsystems through communication topology. Accordingly, numerous classes of network systems with multiple layers of communication can be effectively dealt in hierarchical framework [1, 2]. For example, [3] conducted a hierarchical consensus problem for MAS with low-rank interconnection by investigating the eigenvalue distribution for achieving rapid consensus. Continuing this research direction, a hierarchical network system where some undesirable eigenvalues of local interconnection matrix were selectively moved by proposing low-rank intergroup connection approach in [4]. Subsequently, [5] designed an output consensus in a hierarchical network system employing eigenvector-based interlayer connections.

Many works have proposed systematic synthesis for MAS. LQR design is an effective method to develop a systematic design procedure for MAS. For instance, [6] used a semigroup Kronecker product in order to algebraically characterize hierarchical control problem of MAS utilizing LQR performance criterion. Moreover, [7] proposed a design of distributed control for interconnected system consisting of two layers to guarantee stability of MAS using decentralized controller in lower-layer and to improve the performance with distributed cooperative controller in upper layer. Furthermore, the research [1] and [8] proposed hierarchical decentralized controllers for homogeneous and heterogeneous MAS, respectively. In these approaches, both the global and local objectives were obtained by choosing appropriate weighting matrices of LQR design in the local layer. Recently, [9] proposed hierarchically decentralized control for MAS employing aggregation. "Global/local shared model set" was given to represent the trade-off between global and local performance and to illustrate information sharing among local and global controllers. Following this research line, a practical design of multi-motored wheel electric vehicle was demonstrated in [10]. Furthermore, stability and performance of four wheel electrical vehicle cast in hierarchical structure control containing two layers was studied in [11].

Due to the intensive theoretical studies and their applications, consensus is an important issue in MAS [12, 13]. Depending on the number of leaders in MAS, consensus is normally categorized into either leaderless consensus problem or leader-follower consensus problem. The first problem does not have leader, whereas the later has leader [13, 14]. Moreover, uncertainty caused by an inexact model of MAS dynamics is an important issue for the robust control design. Consequently, the article [15] investigates robust design of uncertain leader-follower consensus controllability and observability by extending the results in [16]. It is applicable to directed and switching graph. The proposed method [15] could treat unstable systems, which was an advantage when compared with other studies. Nevertheless, there is no explicit algorithm for solving Riccati inequality provided in [15].

To our best knowledge, there are a few research works on the design of robust consensus for MAS. Motivated by [2], this paper aims to propose a systematic design of robust LQR leader-follower consensus for uncertain MAS. In particular, we consider the class of MAS with parametric uncertainty in both leader and followers. The proposed control design involves two main elements to implement information exchange among a leader and followers and to capture a desired information structure in MAS. We extend the consensus MAS framework for nominal MAS in [2] to consider uncertain MAS. The information structure of MAS having some desirable constraint can be maintained. The proposed design transforms a low-rank Riccati inequality from the original consensus problem to the stability of a new disagreement error system. The new design condition is equivalent to solving a convex optimization formulated by a linear matrix inequality (LMI) for attaining a maximum bound of model uncertainty. The proposed design overcomes previous drawbacks in [15]. The design optimization can be effectively solved using available solvers, such as SDPT3 with CVX package [17].

The main contributions of this paper are twofold. Firstly, the proposed design is applicable for a robust leader-follower consensus MAS with bounded parametric uncertainty in [15]. It can guarantee the robustness of MAS against the state perturbation. Secondly, numerical results demonstrate that the proposed robust control can guarantee much larger admissible uncertainty bound than the nominal controller and the consensus is achieved with faster speed.

The organization of this paper is as follows. Section 2 describes the model of leader-follower uncertain MAS. In Section 3, a robust consensus of cooperative LQR using state feedback control law is derived. Section 4 provides numerical examples. Finally, Section 5 gives conclusions.

2. Problem Formulation

The following notations are used in this paper. $\mathbb{R}^{n\times n}$ is the set of real $n \times n$ matrix. Moreover, I_n and $\mathbf{1}_n$ represent $n \times n$ identity matrix and $n \times 1$ vector with all elements 1, respectively. Next, $\mathbf{0}$ denotes a matrix with all elements 0 of appropriate dimension. Furthermore, for given symmetric matrix H, $H \succ \mathbf{0}$ ($H \succeq \mathbf{0}$) means that H is positive definiteness (positive semi-definiteness). Similarly, $H \prec \mathbf{0}$ or $H \preceq \mathbf{0}$ indicates that H is negative definiteness or negative semi-definiteness. Lastly, $\otimes$ stands for the Kronecker product.

We first give a review of basic graph theory and terminologies [18]. The information structure in MAS can be represented by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, in which each node and each edge represent an agent and a link to connect two agents, respectively. The set of vertices and edges of $\mathcal{G}$ are denoted by $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and $\mathcal{E} = \{(v_i, v_j) : v_i, v_j \in \mathcal{V}\} \subseteq \mathcal{V} \times \mathcal{V}$. If agent i connects agent j, then there exists an edge $e_{ij} \in \mathcal{E}$. The symbol $\mathcal{N}_i \triangleq \{j : e_{ij} \in \mathcal{E}\}$ is used to represent the set of neighboring vertices (agents) of agent i. Let a_{ij} be an element of the adjacency matrix $\mathcal{A}$ of graph $\mathcal{G}$ where $a_{ij} > 0$ if $e_{ij} \in \mathcal{E}$ and $a_{ij} = 0$ if $e_{ij} \notin \mathcal{E}$. Moreover, there is no edge used to connect a node with itself, i.e., $a_{ii} = 0$. The in-degree of vertex v_i is represented by $\deg_i^{\text{in}} \triangleq \sum_{j\in\mathcal{N}_i} a_{ij}$. Similarly, the out-degree of vertex v_i is represented by $\deg_i^{\text{out}} \triangleq \sum_{j\in\mathcal{N}_i} a_{ji}$. Thus, $\mathcal{D} = \text{diag}\{\deg_i^{\text{in}}\}_{i=1,\ldots,N}$ indicates an in-degree matrix of $\mathcal{G}$. Then, the Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ is obtained by $\mathcal{L} = \mathcal{D} - \mathcal{A}$. $\mathcal{L}$ has a zero eigenvalue with the associated eigenvector $\mathbf{1}_N$. If $\deg_i^{\text{in}} = \deg_i^{\text{out}}, i = 1, \ldots N$, then $\mathcal{G}$ is a balanced graph. Graph $\mathcal{G}$ is undirected if and only if $a_{ij} = a_{ji}, \forall i, j = 1 \ldots N$ and hence satisfies $\mathcal{L} = \mathcal{L}^T \succeq \mathbf{0}$. Moreover, an undirected graph $\mathcal{G}$ is connected if there exists a path between any two vertexes.

The dynamic of each follower in MAS with parametric uncertainty is expressed by the state-space model

$$\begin{cases} \dot{x}_i(t) = (A + V\Delta(t)W)x_i(t) + Bu_i(t) \\ y_i(t) = Cx_i(t) \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^q$, $y_i(t) \in \mathbb{R}^m$ represent the state vector, control input vector, measured output, of the i-th follower. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times q}$, $C \in \mathbb{R}^{m\times n}$, $0 < q \leq n$ denote system matrix, input matrix, and output matrix, respectively. $\Delta(t)$ is an unknown time-varying uncertain matrix satisfying a bounded condition $\Delta(t)^T\Delta(t) \preceq \theta^2 I_n$ or $\|\Delta(t)\|_2 \leq \theta$. $V \in \mathbb{R}^{n\times n}$ and $W \in \mathbb{R}^{n\times n}$ are known matrices to describe the effect of uncertainty $\Delta(t)$ to dynamic matrix A in (1) [19].

The MAS containing N identical followers can be represented in a compact form

$$\begin{cases} \dot{x}(t) = (I_N \otimes (A + V\Delta(t)W))x(t) \\ \qquad + (I_N \otimes B)u(t) \\ y(t) = (I_N \otimes C)x(t), \end{cases} \tag{2}$$

where

$$\begin{aligned} x(t) &\triangleq \begin{bmatrix} x_1(t)^T & \ldots & x_N(t)^T \end{bmatrix}^T, \\ u(t) &\triangleq \begin{bmatrix} u_1(t)^T & \ldots & u_N(t)^T \end{bmatrix}^T, \\ y(t) &\triangleq \begin{bmatrix} y_1(t)^T & \ldots & y_N(t)^T \end{bmatrix}^T. \end{aligned}$$

We make the following assumptions for the MAS.

Assumption 1. (A, B) *is controllable.*

Assumption 2. *Graph $\mathcal{G}$ is fixed, undirected and connected, and there exists at least one follower connecting to the leader r.*

The assumptions 1 and 2 are used to guarantee the existence of controller. The followers in a small subset of nodes in the graph $\mathcal{G}$ observe the leader whose dynamic model is described by

$$\begin{cases} \dot{x}_r(t) = (A + V\Delta(t)W)x_r(t) \\ y_r(t) = Cx_r(t) \end{cases}, \tag{3}$$

where $x_r(t) \in \mathbb{R}^n$, $y_r(t) \in \mathbb{R}^m$ denote the state vector, measured output of the leader. Hence, there exists an edge (v_r, v_i) if the follower i is connected to leader r with weighting gain $g_i > 0$. Otherwise, $g_i = 0$. Then the node i is called pinned node. Let $\Sigma = \text{diag}\{g_1, \ldots, g_N\}$ be the pinning matrix. If $\Sigma = \mathbf{0}$, it is considered as the leaderless consensus problem [2, 14]. Consequently, there exists a new graph $\bar{\mathcal{G}} \in \mathbb{R}^{(N+1)\times(N+1)}$ whose structure matrix is $\mathcal{H} = \mathcal{L} + \Sigma \in \mathbb{R}^{N\times N}$ to represent a combination network of MAS with a leader.

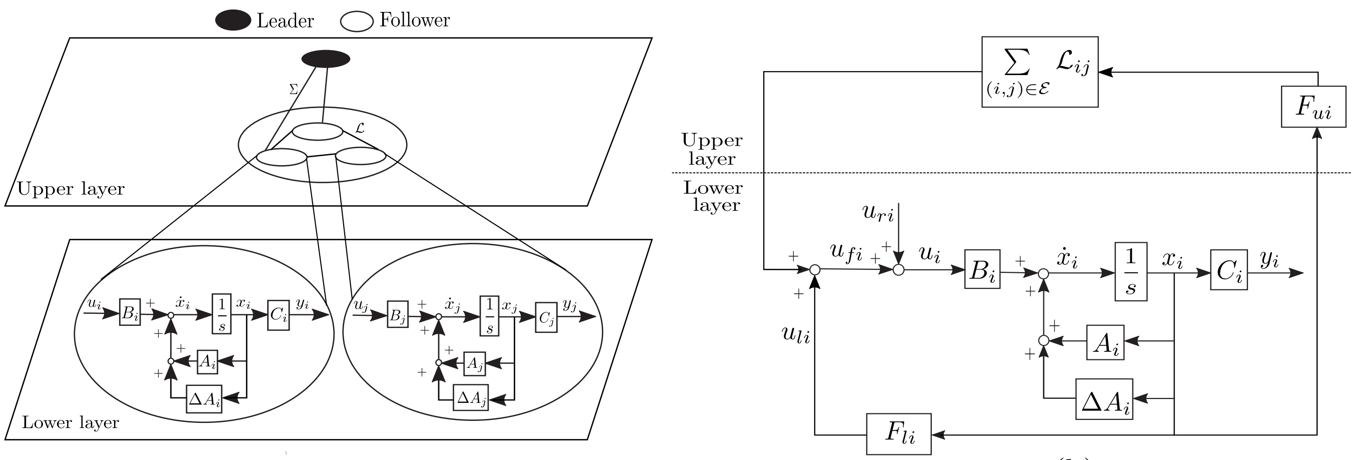
The MAS including N followers whose dynamical model are described by (1) reaches state consensus with respect to the leader (3) if the following conditions hold [20]

$$\lim_{t\to\infty} \|x_i(t) - x_r(t)\| = \mathbf{0}, \forall i = 1, \ldots, N, \tag{4}$$

for any initial conditions $x_i(0)$ and $x_r(0)$.

For simplification, we focus on a two-layer hierarchical structure system that is adopted with modification from [2, 4, 6] and then is depicted in Figure 1 (a). That is to say we want to have a local control component in the lower layer or physical layer and a cooperative/global control component in the upper layer or cyber layer where the agents interact with each other. In fact, this hierarchical structure can become much more complex with many more layers if we have subgroups

DOI:10.4186/ej.2020.24.3.169



(a) (b)

Fig. 1. (a) A two-layer hierarchical structure system; (b) Hierarchical optimal cooperative LQR control structure of i-th follower.

of agents inside the considering MAS, or if we have different time scales in the system.

The main objective of this work is to design a cooperative LQR control for uncertain MAS (2) such that a constraint on information exchange among followers based on graph $\mathcal{G}$ is satisfied, and followers are cooperative in tracking the leader (3). The control input of network system consists of two terms

$$u(t) = u_f(t) + u_r(t), \tag{5}$$

where $u_f(t) = -F_c x(t)$ and $u_r(t) = -(\Sigma \otimes K)(x(t) - \mathbf{1}_N \otimes x_r(t))$ in which K is the coupling matrix from leader to followers denote the control input generated by feedback controller, and the input from the leader, respectively. The control structure is adopted with modification from [1, 2]. It is then illustrated in the Figure 1 (b). If the feedback gain F_c belongs to the following class, then a constraint on information exchange among agents can be ensured [2]

$$\mathcal{F} \triangleq \{F_c \in \mathbb{R}^{Nq \times Nn} | F_c = I_N \otimes F_l + \mathcal{L} \otimes F_u\}, \tag{6}$$

where $F_l, F_u \in \mathbb{R}^{q \times n}$ indicate the local, and the global feedback gain.

The following lemma is utilized in the sequel.

Lemma 1. *[2, 14] The diagonal element of the pinning matrix Σ has at least one non-zero element and all eigenvalues of the structure matrix $\mathcal{H}$ have positive real part. Furthermore, the structure matrix $\mathcal{H}$ is symmetric positive definite since graph $\mathcal{G}$ is assumed to be undirected.*

3. Robust Consensus Design

In this section, we consider a robust leader-follower consensus problem for uncertain MAS described by fixed and undirected topology. There are important differences compared to the previous studies [15] and [2]. Firstly, we extend the result of [2] which only studies the dynamic of nominal followers in MAS and leader to robust design for uncertain MAS. Secondly, we consider MAS represented by an undirected and fixed graph, whereas [15] is applicable for directed and switching topology. Thirdly, we provide a systematic algorithm using LMI formulation to solve the Riccati equation.

The control structure employs the state feedback given in (5). We define the local and global performance functions of MAS as follows:

$$J = J_{x,l} + J_{x,g} + J_u = \int_0^\infty (x^T Q x + u^T R u) dt, \tag{7}$$

where
$J_{x,l} = \int_0^\infty x^T (I_N \otimes Q_1) x dt$ is local performance index,
$J_{x,g} = \int_0^\infty x^T (\mathcal{L} \otimes Q_2) x dt$ is global performance index,
and
$J_u = \int_0^\infty u^T R u dt$ is control input penalty. The Laplacian matrix $\mathcal{L} \in \mathbb{R}^{N \times N}$ indicates the information exchange among agents, agents' relative information, and $Q_1 \in \mathbb{R}^{n \times n}$, $Q_2 \in \mathbb{R}^{n \times n}$, $Q_1 \succeq \mathbf{0}$, $Q_2 \succeq \mathbf{0}$, $R \in \mathbb{R}^{Nq \times Nq}$, $R \succ \mathbf{0}$. The weighting matrices in (7) are chosen as follows [1]:

$$Q = I_N \otimes Q_1 + \mathcal{L} \otimes Q_2,$$
$$R^{-1} = I_N \otimes R_1 + \mathcal{L} \otimes R_2,$$

where $R_1 \in \mathbb{R}^{q \times q}$, $R_1 \succ \mathbf{0}$ and $R_2 \in \mathbb{R}^{q \times q}$, $R_2 \succ \mathbf{0}$ represent weighting matrices in control input penalty of the local and global performance indexes, respectively.

An optimal LQR controller involving both feedback gains F_l and F_u can be obtained by minimizing J in (7). A controller with only global cooperation F_u term is obtained by removing the local term F_l to have consensus for the considering MAS. Accordingly, it is so-called cooperative LQR controller and can be rewritten as follows:

$$F_c = \mathcal{L} \otimes F_u. \tag{8}$$

As pointed out in [2], this controller also belongs to the class $\mathcal{F}$ defined in (6). Also, it makes leader-follower MAS consensus.

Therefore, a robust cooperative LQR controller can be expressed by

$$\begin{aligned} u(t) =& -(\mathcal{L} \otimes F_u)x(t) - (\Sigma \otimes K)(x(t) \\ & - \mathbf{1}_N \otimes x_r(t)). \end{aligned} \tag{9}$$

The following theorem shows the main result of current paper.

Theorem 1. *The state of homogeneous uncertain MAS* (1) *with the controller* (9) *reaches consensus with respect to reference states* (3) *if there exist matrix variable* $X \in \mathbb{R}^{N\times N}$ *and scalar variable* β *that are the solution of the following LMI problem*

$$\begin{aligned} & \textit{minimize } \beta \\ & \textit{subject to} \\ & \beta > 0, \\ & X = X^T \succ \mathbf{0}, \\ & \begin{bmatrix} AX+XA^T-2\alpha BR_1B^T & XW^T & V \\ WX & -I_n & \mathbf{0} \\ V^T & \mathbf{0} & -\beta I_n \end{bmatrix} \preceq \mathbf{0}, \end{aligned} \tag{10}$$

where α *represents the smallest real part of eigenvalue of structure matrix* $\mathcal{H}$, *and the following conditions hold.*

C1. $\Sigma \neq \mathbf{0} \in \mathbb{R}^{N\times N}$.

C2. $R_2 = R_1 + S$ *with* $S \succeq \mathbf{0}$.

C3. $F_u = K = R_2 B^T P_1$.

Proof. Let the global synchronization error (disagreement vector) be

$$e(t) = x(t) - \mathbf{1}_N \otimes x_r(t). \tag{11}$$

Substituting (11) into (9) and using Conditions C3, the state feedback controller now becomes

$$u(t) = -\mathcal{L} \otimes (R_2B^TP_1)x(t) - (\Sigma \otimes K)e(t). \tag{12}$$

Taking derivative of $e(t)$ in (11) along the trajectory (2) and (3) obtains

$$\begin{aligned} \dot{e}(t) &= \dot{x}(t) - \mathbf{1}_N\dot{x}_r(t) \\ &= (I_N \otimes (A + V\Delta(t)W))x(t) + (I_N \otimes B)u(t) \\ &\quad - \mathbf{1}_N \otimes ((A + V\Delta(t)W)x_r(t)) \\ &= (I_N \otimes (A + V\Delta(t)W) - \mathcal{L} \otimes (BR_2B^TP_1) \\ &\quad - \Sigma \otimes (BK))e(t) - (\mathcal{L}\mathbf{1}_N) \otimes (BR_2B^TP_1)x_r(t) \\ &= (I_N \otimes (A + V\Delta(t)W) - (\mathcal{L} \\ &\quad + \Sigma) \otimes (BR_2B^TP_1))e(t). \end{aligned} \tag{13}$$

It is noted that $\mathcal{L}\mathbf{1}_N = \mathbf{0}$. Denote

$$\begin{aligned} A_e(t) =& I_N \otimes (A + V\Delta(t)W) \\ & - (\mathcal{L} + \Sigma) \otimes (BR_2B^TP_1). \end{aligned} \tag{14}$$

Then, we can rewrite (14) as follows:

$$A_e(t) = I_N \otimes (A + V\Delta(t)W) - \mathcal{H} \otimes (BK).$$

Hence, the leader-followers consensus synthesis is equivalent to designing matrices $\mathcal{L}$, Σ, $\mathcal{H}$, and K such that the system (13) rewritten in the following form

$$\dot{e}(t) = A_e(t)e(t), \tag{15}$$

is stable, or matrix $A_e(t)$ is Hurwitz. If Lemma 1 and condition C1 are satisfied, there exists an unitary matrix $U \in \mathbb{R}^{N\times N}$ such that $U^T\mathcal{H}U = \Gamma$ where $\Gamma = \mathrm{diag}\{\gamma_1, \dots, \gamma_N\}$ whose diagonal elements are eigenvalues of $\mathcal{H}$. Therefore, multiplying both sides of system (13) with $(U^T \otimes I_n)$ and let $\tilde{e}(t) = (U^T \otimes I_n)e(t)$ yields

$$\dot{\tilde{e}}(t) = (I_N \otimes (A + V\Delta(t)W) - \Gamma \otimes (BK))\tilde{e}(t). \tag{16}$$

Obviously, the system (16) can be decomposed into N independent subsystems. Hence, the stability analysis problem of (16) is equivalent to that of its diagonal subsystems

$$\begin{aligned} \dot{\tilde{e}}_i(t) &= [(A + V\Delta(t)W) - \gamma_i(BK)]\tilde{e}_i(t), \\ & i = 1, \dots, N. \end{aligned} \tag{17}$$

In addition, (17) can be rewritten in the following form

$$\begin{aligned} \dot{\tilde{e}}_i(t) &= (A - \gamma_i(BK))\tilde{e}_i(t) + Vq_i(t), \\ & i = 1, \dots, N, \end{aligned} \tag{18}$$

where $q_i(t) = \Delta(t)W\tilde{e}_i(t)$. The following non-negative function is considered

$$V(\tilde{e}_i(t)) = \tilde{e}_i^TP_1\tilde{e}_i(t), \tag{19}$$

where $P_1 \succ \mathbf{0}$ is the solution of the following Riccati inequality

$$\begin{aligned} & P_1A + A^TP_1 - 2\alpha P_1BR_1B^TP_1 \\ & \quad + \theta^2P_1VV^TP_1 + W^TW \preceq \mathbf{0}. \end{aligned} \tag{20}$$

Taking derivative of $V(\tilde{e}_i(t))$ along the time trajectory (17) yields

$$\begin{aligned} \dot{V}(\tilde{e}_i(t)) =& \tilde{e}_i^T(P_1A + A^TP_1 - 2\gamma_iP_1BK)\tilde{e}_i(t) \\ & + 2\tilde{e}_i^TP_1Vq_i(t). \end{aligned} \tag{21}$$

It follows from $\alpha = \min\{\gamma_i\}_{i=1,\ldots,N}$ that $P_1A+A^TP_1-2\gamma_iP_1BK \preceq P_1A + A^TP_1 - 2\alpha P_1BK$. Then, we can show that (21) results in

$$\begin{aligned}
&\dot{V}(\tilde{e}_i(t)) \\
&\le \tilde{e}_i^T(P_1A + A^TP_1 - 2\alpha P_1BK)\tilde{e}_i(t) + 2\tilde{e}_i^TP_1Vq_i(t), \\
&= \begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix}^T \begin{bmatrix} P_1A+A^TP_1-2\alpha P_1BK & P_1V \\ V^TP_1 & \mathbf{0}\end{bmatrix} \begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix}.
\end{aligned} \tag{22}$$

If $\Delta(t)^T\Delta(t) \preceq \theta^2 I_n$, then $q_i(t)^Tq_i(t) = \tilde{e}_i(t)^TW^T\Delta(t)^T\Delta(t)W\tilde{e}_i(t) \le \tilde{e}_i(t)^TW^T\theta^2I_nW\tilde{e}_i(t)$ and hence the following LMIs hold [15]

$$\begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix}^T \begin{bmatrix} W^TW & \mathbf{0} \\ \mathbf{0} & -\theta^{-2}I_n\end{bmatrix} \begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix} \ge 0, \forall i = 1,\ldots,N. \tag{23}$$

It follows from (22) and (23) and by deducing from [15] that

$$\begin{aligned}
&\dot{V}(\tilde{e}_i(t)) \\
&\le \begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix}^T \begin{bmatrix} P_1A + A^TP_1 - 2\alpha P_1BK & P_1V \\ V^TP_1 & \mathbf{0}\end{bmatrix} \begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix} \\
&+ \begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix}^T \begin{bmatrix} W^TW & \mathbf{0} \\ \mathbf{0} & -\theta^{-2}I_n\end{bmatrix} \begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix}, \forall i = 1,\ldots,N \\
&= \begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix}^T \begin{bmatrix} P_1A+A^TP_1-2\alpha P_1BR_2B^TP_1+W^TW & P_1V \\ V^TP_1 & -\theta^{-2}I_n\end{bmatrix} \\
&\times \begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix}, \forall i = 1,\ldots,N.
\end{aligned} \tag{24}$$

When the condition C2 is satisfied, (24) is equivalent to

$$\begin{aligned}
&\dot{V}(\tilde{e}_i(t)) \le \begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix}^T \\
&\times \left\{ \begin{bmatrix} P_1A+A^TP_1-2\alpha P_1BR_1B^TP_1+W^TW & P_1V \\ V^TP_1 & -\theta^{-2}I_n\end{bmatrix} \right. \\
&\left. + \begin{bmatrix} P_1B \\ \mathbf{0}\end{bmatrix} (-2\alpha S) \begin{bmatrix} B^TP_1 & \mathbf{0}\end{bmatrix} \right\} \times \begin{bmatrix}\tilde{e}_i(t)\\ q_i(t)\end{bmatrix}, \\
&\forall i = 1,\ldots,N.
\end{aligned} \tag{25}$$

By utilizing the Schur's complement [21], the Riccati inequality (20) can be rewritten as

$$\begin{bmatrix} P_1A+A^TP_1-2\alpha P_1BR_1B^TP_1+W^TW & P_1V \\ V^TP_1 & -\theta^{-2}I_n\end{bmatrix} \preceq \mathbf{0}. \tag{26}$$

It can be shown that

$$\begin{aligned}
&\begin{bmatrix} P_1B \\ \mathbf{0}\end{bmatrix} [-2\alpha S] \begin{bmatrix} P_1B & \mathbf{0}\end{bmatrix} \\
&\quad = \begin{bmatrix} B^TP_1 & \mathbf{0}\end{bmatrix}^T [-2\alpha S] \begin{bmatrix} B^TP_1 & \mathbf{0}\end{bmatrix} \preceq \mathbf{0}.
\end{aligned} \tag{27}$$

Let

$$E = \begin{bmatrix} B^TP_1 & \mathbf{0}\end{bmatrix}, F = -2\alpha S. \tag{28}$$

Similarly to [21], because $S \succeq \mathbf{0}$ in C2, $F \preceq \mathbf{0}$, with any vector $p \in \mathbb{R}^n \neq \mathbf{0}$, $p^TEp \le 0$. Moreover, $Ep \neq \mathbf{0}$ if E is full column rank, i.e. $\mathcal{N}(E) = \{\mathbf{0}\}$. Accordingly, $(Ep)^TF(Ep) \le 0$, or it is equivalent $p^TE^TFEp \le 0$. Thus, (27) holds. Based on (25), (26), (27), we obtain

$$\dot{V}(\tilde{e}_i(t)) \le 0. \tag{29}$$

It means that all independent subsystems (16) are stable. Consequently, the leader-follower consensus of the MAS (1) is achieved. Furthermore, we show how (10) is derived. Let $X \triangleq P_1^{-1}$. Riccati inequality (20) is equivalent to the following LMI condition.

$$\begin{aligned}
&X = X^T \succ \mathbf{0}, \\
&\begin{bmatrix} AX+XA^T-2\alpha BR_1B^T+\theta^2VV^T & XW^T \\ WX & -I_n\end{bmatrix} \preceq \mathbf{0},
\end{aligned} \tag{30}$$

Decomposing (30) gives

$$\begin{aligned}
&X = X^T \succ \mathbf{0}, \\
&\begin{bmatrix} AX+XA^T-2\alpha BR_1B^T & XW^T \\ WX & -I_n\end{bmatrix} + \theta^2 \begin{bmatrix} V \\ \mathbf{0}\end{bmatrix} \begin{bmatrix} V^T & \mathbf{0}\end{bmatrix} \preceq \mathbf{0},
\end{aligned} \tag{31}$$

Then, applying Schur's complement [21] to (31), we obtain

$$\begin{aligned}
&X = X^T \succ \mathbf{0}, \\
&\begin{bmatrix} AX+XA^T-2\alpha BR_1B^T & XW^T & V \\ WX & -I_n & \mathbf{0} \\ V^T & \mathbf{0} & -\frac{1}{\theta^2}I_n\end{bmatrix} \preceq \mathbf{0}.
\end{aligned} \tag{32}$$

Let $\beta = \frac{1}{\theta^2}$, then (10) holds. Therefore, $X = X^T \succ \mathbf{0}$ and a maximum admissible bound of uncertainty $\theta_{\max}$ associated with $\beta_{\min}$ satisfying Riccati inequality (20) can be obtained by solving the LMI (10). ■

A consensus design of robust cooperative LQR control contains 5 steps as follows:

S1. Local setting: Choose a matrix $R_1 \in \mathbb{R}^{q\times q}$, $R_1 \succ \mathbf{0}$.

S2. Global setting: Derive Laplacian matrix $\mathcal{L} \in \mathbb{R}^{N\times N}$ of graph $\mathcal{G}$.

S3. Weighting matrices setting: Derive a pinning matrix $\Sigma \in \mathbb{R}^{N\times N}$ and then compute a structure matrix $\mathcal{H}$ and α. Then, choose a matrix $R_2 \in \mathbb{R}^{q\times q}$, $R_2 \succ \mathbf{0}$ as condition C2 of Theorem 1.

S4. LMI solution: Solve LMI (10) to obtain solution in term of (X, β).

S5. Control calculation: Compute controller (9).

The main result of this work is the LMI formulation (10), which is equivalent to the Riccati inequality (20). In order to clearly demonstrate the advantage of the proposed design, we will compare the results of robust consensus with that of nominal cooperative LQR controller [2] under the same assumptions. More specifically, we first find the maximum admissible bound of uncertainty that nominal control [2] can guarantee stability of system (1). Then, we compare with the maximum admissible bound of uncertainty obtained by the robust cooperative control. To this end, let us explain how to compute the maximum admissible bound of the uncertainty.

Consider the nominal MAS, namely, Eq. (1) where $\Delta(t) = \mathbf{0}$. To design a consensus of nominal MAS, Riccati equation was first reported in [2] and given as follows:

$$\begin{aligned} &P_{1\text{nom}}A + A^T P_{1\text{nom}} \\ &- P_{1\text{nom}} B R_1 B^T P_{1\text{nom}} + Q_1 = \mathbf{0}, \end{aligned} \tag{33}$$

where $P_{1\text{nom}} \succ \mathbf{0}$ and $Q_1 \succ \mathbf{0}$. The nominal feedback LQR controller has the form

$$K = R_2 B^T P_{1\text{nom}}. \tag{34}$$

Next, we wish to find $\tilde{P} \in \mathbb{R}^{n\times n}$ of Lyapunov function candidate $V(\tilde{e}_i(t)) = \tilde{e}_i^T \tilde{P} \tilde{e}_i(t)$ which guarantees the quadratic stability of uncertain MAS (15). In addition, we aim to compute the maximum admissible bound of the uncertainty when applying the nominal LQR control. Define $\beta = \frac{1}{\theta^2}$. It can be shown that the maximum admissible bound of the uncertainty can be determined by solving the following LMI.

$$\begin{aligned} &\text{minimize } \beta \\ &\text{subject to} \\ &\beta > 0, \\ &Z = Z^T \succ \mathbf{0}, \\ &\begin{bmatrix} (A-\alpha BK)^T Z + Z(A-\alpha BK) + W^T W & ZV \\ V^T Z & -\beta I_n \end{bmatrix} \preceq \mathbf{0}. \end{aligned} \tag{35}$$

Note that we apply S-procedure [22] to derive (35). The sufficient condition $\dot{V}(\tilde{e}_i(t)) \leq 0$ along the trajectory(18) is satisfied if and only if there exists matrix $\tilde{P} \succeq \mathbf{0}$ and a positive scalar τ such that

$$\begin{bmatrix} (A-\gamma_i BK)^T \tilde{P} + \tilde{P}(A-\gamma_i BK) & \tilde{P}V \\ V^T \tilde{P} & \mathbf{0} \end{bmatrix} \preceq \tau \begin{bmatrix} -W^T W & \mathbf{0} \\ \mathbf{0} & \frac{1}{\theta^2} I_n \end{bmatrix}, \quad \forall i = 1, \ldots, N. \tag{36}$$

It implies that

$$\begin{bmatrix} (A-\gamma_i BK)^T \tilde{P} + \tilde{P}(A-\gamma_i BK) + \tau W^T W & \tilde{P}V \\ V^T \tilde{P} & -\tau \frac{1}{\theta^2} I_n \end{bmatrix} \preceq \mathbf{0}, \quad \forall i = 1, \ldots, N. \tag{37}$$

Using Schur's complement [21] to (37), we have

$$\begin{aligned} &(A-\gamma_i BK)^T \tilde{P} + \tilde{P}(A-\gamma_i BK) \\ &+\tau W^T W + \tilde{P}V \frac{\theta^2}{\tau} I_n V^T \tilde{P} \preceq \mathbf{0}, \\ &\forall i = 1, \ldots, N. \end{aligned} \tag{38}$$

Expanding (38) and then isolating all terms containing γ_i yields

$$\begin{aligned} &A^T \tilde{P} + \tilde{P}A - \gamma_i((BK)^T \tilde{P} + \tilde{P}BK) \\ &+\tau W^T W + \tilde{P}V \frac{\theta^2}{\tau} I_n V^T \tilde{P} \preceq \mathbf{0}, \\ &\forall i = 1, \ldots, N. \end{aligned} \tag{39}$$

By defining $Z = \frac{1}{\tau}\tilde{P}$ in (39), it follows that

$$\begin{aligned} &A^T Z + ZA + W^T W + ZV\theta^2 I_n V^T Z \\ &\preceq \gamma_i((BK)^T Z + ZBK), \\ &\forall i = 1, \ldots, N. \end{aligned} \tag{40}$$

Recall that α be the smallest eigenvalue of the structure matrix $\mathcal{H}$, i.e., $\alpha = \min\{\gamma_i\}_{i=1,\ldots,N}$. Thus, (40) is equivalent to

$$\begin{aligned} &A^T Z + ZA + W^T W + ZV\theta^2 I_n V^T Z \\ &\preceq \alpha((BK)^T Z + ZBK). \end{aligned} \tag{41}$$

Let $\beta = \frac{1}{\theta^2}$. Applying Schur's complement [21] for (41) obtains (35). Once LMI (35) is feasible, the uncertain MAS (15) is quadratically stable in the presence of bounded uncertainty θ. It is noted that (35) can be considered as a special case of (10).

4. Numerical Examples

To illustrate the effectiveness of the proposed robust consensus design, we consider three homogeneous following agents and a leading agent whose system matrices are given by

$$A = \begin{bmatrix} -1 & 2 \\ 1 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}, V = W = I_2,$$

$\Delta(t) = \text{diag}\{\theta\sin(\text{t}), \theta\cos(\text{t})\}$ where θ is found from (10). Note that (A, B) is controllable, and A is a stable matrix.

$\mathcal{L}$ of graph $\mathcal{G}$ and Σ from Figure 2 are given by

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \Sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

DOI:10.4186/ej.2020.24.3.169

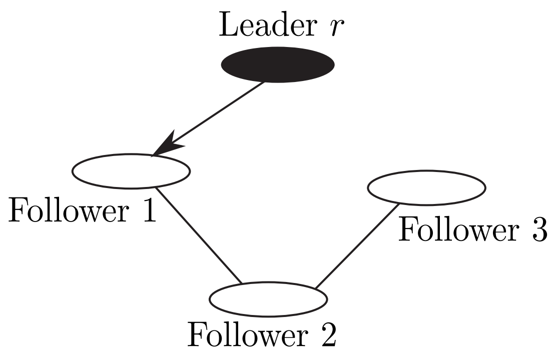


Fig. 2. Undirected graph of given MAS.

Table 1. Comparison of design results of feedback gain and maximum admissible bound of uncertainty.

Control method	K	$\theta_{\max}$
Robust LQR control: Theorem 1	[5.1261 3.8047]	0.9137
Nominal LQR control [2]	[0.7677 0.5433]	0.5096

It means that the agents 1 and 2 connect to agents 2 and 3, respectively, and the leader r connects to only the agent 1. The structure matrix $\mathcal{H}$ and α are computed as

$$\mathcal{H} = \mathcal{L} + \Sigma = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \alpha = 0.1981.$$

Using the proposed design procedure, we select $R_1 = 1$, $Q_1 = I_2$, and $S = 0.1$. Therefore, $R_2 = R_1 + S = 1.1$. Table 1 summarizes the state-feedback gain of two controllers and the corresponding maximum admissible bound of uncertainty $\theta_{\max}$. It is observed that $\theta_{\max}$ obtained from (10) results in high-gain controller (9). A small gain controller can be achieved by selecting θ less than $\theta_{\max}$. Obviously, $\theta_{\max}$ of nominal cooperative LQR controller in [2] is less than that of our proposed controller.

Let us use the value $\theta = 0.9137$ in the simulation. Therefore, the robust cooperative LQR state feedback controller having the form (9) is determined as

$$\begin{aligned} u(t) = \\ -&\begin{bmatrix} 5.1261 & 3.8047 & -5.1261 & -3.8047 & 0 & 0 \\ -5.1261 & -3.8047 & 10.2522 & 7.6094 & -5.1261 & -3.8047 \\ 0 & 0 & -5.1261 & -3.8047 & 5.1261 & 3.8047 \end{bmatrix} x(t) \\ +&\begin{bmatrix} 5.1261 & 3.8047 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} (\mathbf{1}_N \otimes x_r(t)). \end{aligned}$$

On the other hand, the nominal cooperative LQR controller adopted from [2] is given by

$$\begin{aligned} u(t) = \\ -&\begin{bmatrix} 0.7677 & 0.5433 & -0.7677 & -0.5433 & 0 & 0 \\ -0.7677 & -0.5433 & 1.5354 & 1.0865 & -0.7677 & -0.5433 \\ 0 & 0 & -0.7677 & -0.5433 & 0.7677 & 0.5433 \end{bmatrix} x(t) \\ +&\begin{bmatrix} 0.7677 & 0.5433 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} (\mathbf{1}_N \otimes x_r(t)). \end{aligned}$$

Next, initial conditions on the states of the leader and the followers are specified as follows:

$$x_r(0) = \begin{bmatrix} -0.8 & 0.3 \end{bmatrix}^T, x_1(0) = \begin{bmatrix} 0.5 & -1.5 \end{bmatrix}^T,$$
$$x_2(0) = \begin{bmatrix} 0.2 & -0.6 \end{bmatrix}^T, x_3(0) = \begin{bmatrix} -0.5 & 1 \end{bmatrix}^T.$$

Table 1 shows that the proposed controller can assure consensus with wider range of parameter uncertainty than the comparative controller. Hence, the result reveals that the robust control is less conservative than the nominal control. Subsequently, we compare the responses of MAS and control inputs of two controllers to validate the effectiveness of the robust consensus design and to demonstrate advantage in comparison with nominal control design. The simulated response of MAS and control inputs are depicted in Figures 3–4. We observe that the proposed robust cooperative control gives faster convergence of states than that of the nominal cooperative LQR control.

Next, we will show the comparison on the feedback gain and $\theta_{\max}$ when varying the number of agents. We choose the number of agents as $N = 3, 9, 27$ and use the graph similar to the case $N = 3$ depicted in Figure 2. This implies the leader only connects to the 1st follower, and the i-th follower connects with its $i-1$-th and $i+1$-th neighbours. This inter-follower structure is called a path graph. We point out that two matrices $\mathcal{L}$ and $\mathcal{H}$ need to be updated when varying the number of agents. Table 2 shows the calculated feedback gain. It can be seen that feedback gain of robust LQR control is gradually reduced when the number of agents increases. On the other hand, feedback gain of nominal LQR control keeps constant regardless of the number of agents. Moreover, the robust consensus control always gives $\theta_{\max}$ greater than that obtained from the nominal consensus design as shown in Table 3. When the number of agent N is increased, it affects the value of

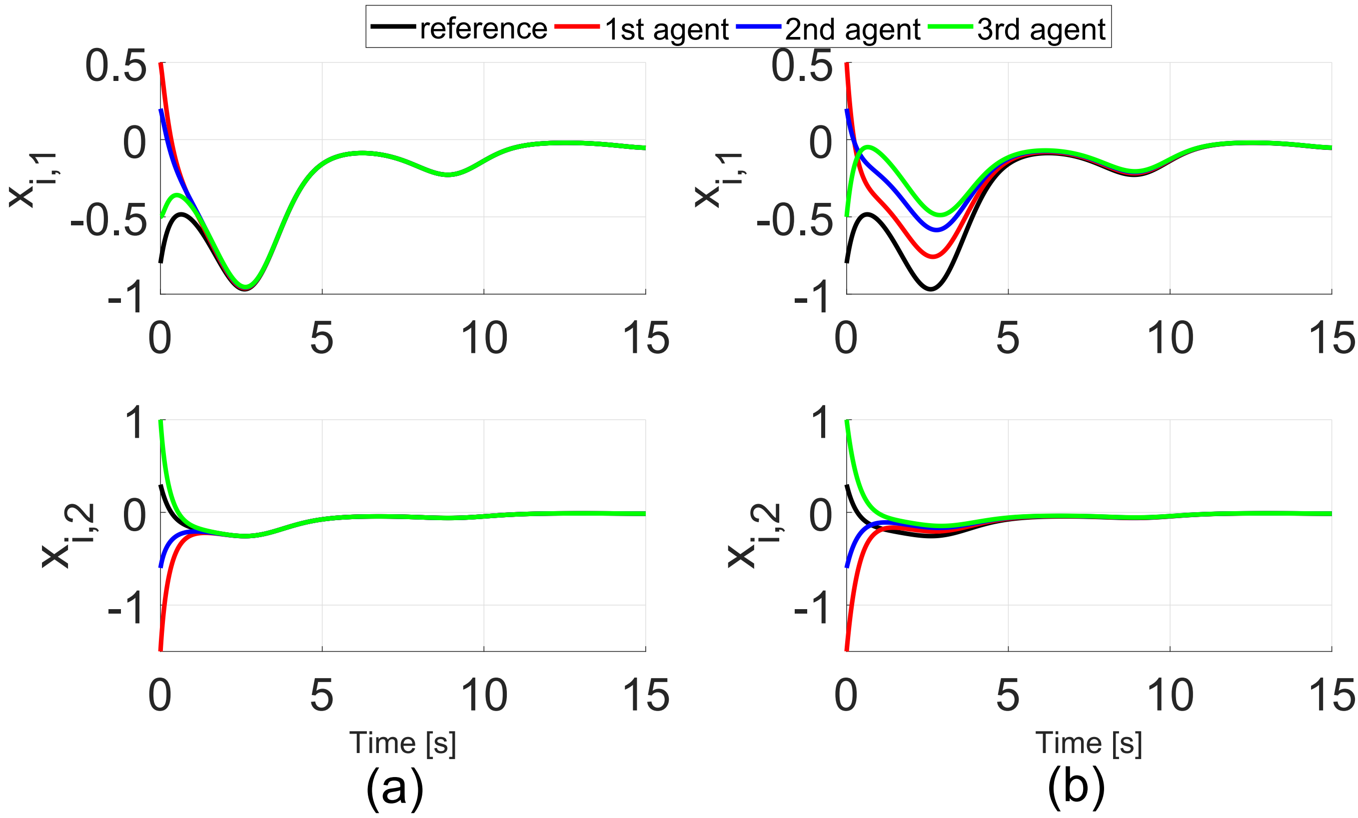


Fig. 3. Response of MAS with respect to the reference: (a) robust cooperative LQR control; (b) nominal cooperative LQR control.

the smallest eigenvalue α of structure matrix $\mathcal{H}$. Subsequently, it results in an increase of $\beta_{\min}$ and a decrease of $\theta_{\max}$. Further investigation is needed to explain the relationship between the number of agents and the structure matrix.

We show through the numerical examples that parametric uncertainty degrades the performance of control system using the nominal control. The simulation also reveals that the proposed design method obtains faster consensus against parametric uncertainty than that of nominal control method. Thus, robust cooperative LQR design is deemed to offer an effective solution to treat uncertain MAS.

5. Conclusions

This paper presents a systematic design for robust LQR consensus of leader-follower homogeneous uncertain MAS. The dynamic model of leader and followers is subjected to parametric uncertainty. The consensus design problem of the original uncertain MAS is transformed to stability analysis of independent subsystems by employing the decomposition approach. We develop the sufficient condition to guarantee the robust stability using the Riccati inequality. Moreover, we apply the Schur's complement to reformulate the Riccati inequality as low-rank LMI which can be effectively solved. We propose the robust cooperative LQR control to achieve state consensus. Numerical results reveal that the proposed robust LQR control has advantage over the nominal LQR control in terms of the guaranteed bound of uncertainty and consensus speed. An ongoing work is to design a robust consensus for non-identical uncertainty of followers and leader.

Acknowledgment

We gratefully acknowledge the research grant from JICA project for AUN/SEED-Net and from Rachadapisek Sompote Fund for Research Unit on Intelligent Control Automation of Process Systems. We also acknowledge the research facilities provided by the Department of Electrical Engineering, Faculty of Engineering, Chulalongkorn University, and the International Institute for Carbon-Neutral Energy Research, Institute of Mathematics for Industry, Kyushu University.

References

[1] D. H. Nguyen and S. Hara, "Hierarchical decentralized stabilization for networked dynamical systems by LQR selective pole shift," in *Proceeding*

DOI:10.4186/ej.2020.24.3.169

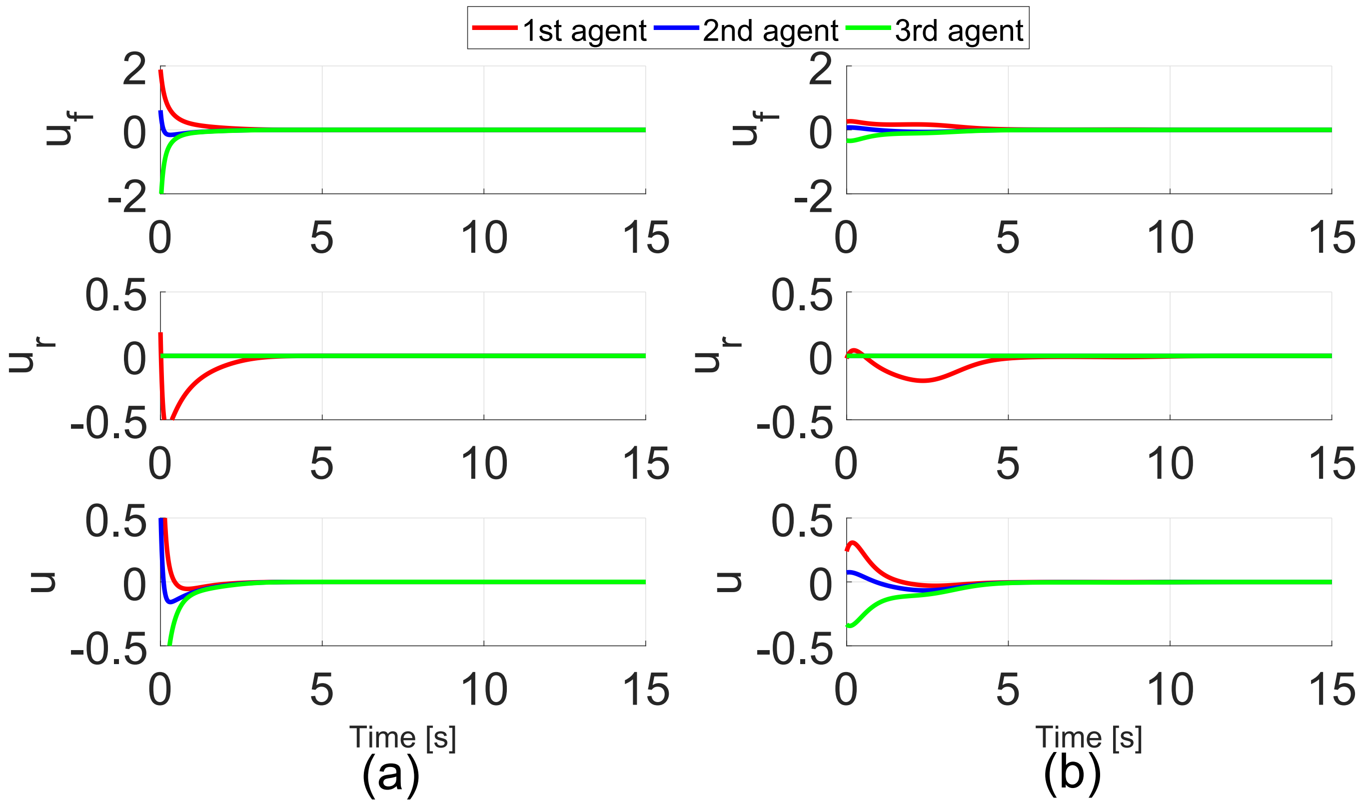


Fig. 4. Control inputs of MAS: (a) robust cooperative LQR control; (b) nominal cooperative LQR control.

Table 2. State feedback gain versus the number of agents.

Control method	Feedback gain K		
	$N = 3$	$N = 9$	$N = 27$
Robust LQR control: Theorem 1	[5.1261 3.8047]	[5.0232 3.7351]	[4.9312 3.6811]
Nominal LQR control [2]	[0.7677 0.5433]	[0.7677 0.5433]	[0.7677 0.5433]

Table 3. Maximum admissible bound of uncertainty versus the number of agents.

Control method	Maximum admissible bound $\theta_{\max}$		
	$N = 3$	$N = 9$	$N = 27$
Robust LQR control: Theorem 1	0.9137	0.6684	0.2822
Nominal LQR control [2]	0.5096	0.3830	0.2629

of 19th World Congress IFAC, Cape Town, South Africa, 24-29 August 2014, pp. 5778–5783.

[2] D. H. Nguyen, "A sub-optimal consensus design for multi-agent systems based on hierarchical LQR," *Automatica*, vol. 55, pp. 88–94, 2015. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0005109815000990

[3] H. Shimizu and S. Hara, "Hierarchical consensus for multi-agent systems with low-rank interconnection," in *Proceeding of (ICROS-SICE) International Joint Conference*, Fukuoka, Japan, 18-21 August 2009, pp. 1063–1067.

[4] D. Tsubakino and S. Hara, "Eigenvector-based intergroup connection of low rank for hierarchical multi-agent dynamical systems," *Systems & Control Letters*, vol. 61, no. 2, pp. 354–361, 2012. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S016769111100291X

[5] N. Fujimori, L. Liu, S. Hara, and D. Tsubakino, "Hierarchical network synthesis for output consensus by eigenvector-based interlayer connections," in *Proceeding of 50th IEEE Conference on Decision and Control and European Control Conference*, Orlando, FL, USA, 12-15 December 2011, pp. 1449–1454.

[6] D. Tsubakino, T. Yoshioka, and S. Hara, "An algebraic approach to hierarchical LQR synthesis for large-scale dynamical systems," in *Proceeding of 9th Asian Control Conference*, Istanbul, Turkey, 23-26 June 2013, pp. 1–6.

[7] A. Gusrialdi, "Performance-oriented communication topology design for distributed control of interconnected systems," *Mathematics of Control, Signals, and Systems*, vol. 25, no. 4, pp. 559–585, 2013. [Online]. Available: https://doi.org/10.1007/s00498-013-0116-4

[8] D. H. Nguyen and S. Hara, "Hierarchical decentralized controller synthesis for heterogeneous multi-agent dynamical systems by LQR," *SICE Journal of Control, Measurement, and System Integration*, vol. 8, no. 4, pp. 295–302, 2015.

[9] S. Hara, K. Tsumura, and B. M. Nguyen, *Hierarchically Decentralized Control for Networked Dynamical Systems with Global and Local Objectives*. Emerging Applications of Control and Systems Theory, Lecture Notes in Control and Information Sciences, Springer, 2018, pp. 179–191. [Online]. Available: https://doi.org/10.1007/978-3-319-67068-3_13

[10] B. M. Nguyen, S. Hara, and K. Tsumura, "Hierarchically decentralized control for in-wheel-motored electric vehicles with global and local objectives," in *Proceeding of 11th Asian Control Conference*, Gold Coast Convention Centre, Australia, December 17-20 2017, pp. 1176–1181.

[11] Z. Wang, Y. Wang, L. Zhang, and M. Liu, "Vehicle stability enhancement through hierarchical control for a four-wheel-independently-actuated electric vehicle," *Energies*, vol. 10, no. 7, 2017. [Online]. Available: http://www.mdpi.com/1996-1073/10/7/947

[12] D. H. Nguyen, T. Narikiyo, and M. Kawanishi, "Robust consensus analysis and design under relative state constraints or uncertainties," *IEEE Transactions on Automatic Control*, vol. 63, no. 6, pp. 1784–1790, 2018.

[13] B. Wang, J. Wang, B. Zhang, H. Lin, X. Li, and H. Wang, "Leader-follower consensus for multi-agent systems with three-layer network framework and dynamic interaction jointly connected topology," *Neurocomputing*, vol. 207, pp. 231–239, 2016. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0925231216303228

[14] T. Feng, H. Zhang, Y. Luo, and H. Liang, "Globally optimal distributed cooperative control for general linear multi-agent systems," *Neurocomputing*, vol. 203, pp. 12–21, 2016. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0925231216302302

[15] W. Ni, X. Wang, and C. Xiong, "Consensus controllability, observability and robust design for leader-following linear multi-agent systems," *Automatica*, vol. 49, no. 7, pp. 2199–2205, 2013. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0005109813001970

[16] W. Ni, X. Wang, and C. Xiong, "Leader-following consensus of multiple linear systems under switching topologies: An averaging method," *Kybernetika*, vol. 48, no. 6, pp. 1194–1210, 2012.

[17] M. Grant, S. Boyd, and Y. Ye, *CVX: Matlab software for disciplined convex programming*. Stanford University, 2017, [Online] Available at http://www.cvxr.com/cvx/.

[18] D. H. Nguyen, "Minimum-rank dynamic output consensus design for heterogeneous nonlinear multi-agent systems," *IEEE Transactions on Control of Network Systems*, vol. 5, no. 1, pp. 105–115, 2018.

[19] C.-H. Lien, "Robust observer-based control of systems with state perturbations via LMI approach," *IEEE Transactions on Automatic Control*, vol. 49, no. 8, pp. 1365–1370, 2004.

[20] W. Ni and D. Cheng, "Leader-following consensus of multi-agent systems under fixed and switching topologies," *Systems & Control Letters*, vol. 59, no. 3, pp. 209–217, 2010. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0167691110000186

[21] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Society for Industrial and Applied Mathematics, 1994. [Online]. Available: https://epubs.siam.org/doi/abs/10.1137/1.9781611970777

[22] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge University Press, 2004.

Tuynh Van Pham received a M.Eng. degree in Electrical Engineering in 2017 from Chulalongkorn University where he is currently working towards a Ph.D. degree in Electrical Engineering. He is a student member of IEEE. His research interests include cooperative control of multi-agent systems, hierarchical structure control, robust control, iterative learning control, and energy management systems.

Dinh Hoa Nguyen received a Ph.D. degree from The University of Tokyo in 2014. Currently, he is an Assistant Professor at Kyushu University, Japan. His research is on modeling, prediction, optimization, and control toward low-carbon and autonomous energy systems, with particular focuses on renewables and distributed energy resources, multi-agent systems, distributed optimization, and smart grid.

David Banjerdpongchai has been with Faculty of Engineering, Chulalongkorn University. Currently, he is a professor of Electrical Engineering, head of Intelligent Control Automation of Process Systems Research Unit, and deputy director of International School of Engineering. He is a senior member of IEEE, a founding chair of IEEE Control Systems Society Thailand Chapter, an executive board member of ECTI Association, and a chair of IEC TC65 Thailand National Committee. He is a TPC co-chair of SICE 2020 and general chair of ICA-SYMP 2021. In addition, he serves as Associate Editor of International Journal of Control Automation Systems and section Editor-in-Chief of ASEAN Engineering Journal. He has published over 150 articles in journal and conference proceedings. His research interests are energy management systems, advanced process control, iterative learning control, and robust control design.