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Article

Production Scheduling with Capacity-Lot Size and Sequence Consideration

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Abstract. The General Lot-sizing and Scheduling Problem (GLSP) is a common problem found in continuous production planning. This problem involves many constraints and decisions including machine capacity, production lot-size, and production sequence. This study proposes a two-phase algorithm for solving large-scale GLSP models. In Phase 1, we generate patterns with a specific batch size and capacity and in Phase 2, based on the patterns selected in Phase 1, we optimize the production allocation. Additionally, the external supplies are included in the formulation to reflect the real situation in business with limited resources. In this work, the justification of the formulation is based on the ability of solving and calculation time. The proposed formulation was tested on eight scenarios. The results show that the proposed formulation is more tractable and is easier to solve than the GLSP.

Keywords: Production scheduling optimization, two-phase optimization, lot-size production scheduling, sequence dependent production scheduling.

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1. Introduction

In manufacturing, an operation requires several planning levels, including strategic, tactical, and operational levels. Strategic planning is the core component of planning because it has to match with the business direction. It involves making decisions such as facility location, facility design, and network design, and often spans long planning horizons. Tactical planning involves balancing the demand and supply of each product family over the medium term. The focus is often profit maximization. By contrast, operation planning often focuses on cost while adhering to the tactical plan from the previous stage.

Production scheduling is one of the most essential tasks in operation planning. It dictates what actions are required on a daily or hourly basis as well as the types and quality of the products required for production, according to the fluctuation in the demand, operation configuration, and production capacity. Often times the decisions include the appropriate inventory levels needed in order to buffer the variations in demand and production uncertainty in each period. Similar to any inventory management situation, the objective here is to minimize inventory investment while meeting the necessary service levels. The production scheduling process can often be formulated as a linear program, which considers demand, inventory carrying cost, and production capacity. Solving this problem is straightforward and effective for use in an actual business setting when the linearity assumption holds.

In Continuous production, the Changeover Cost is an important variable of production planning, as converting from one product to another product might lead to the cost of the operation. Changeover Cost refers to the additional cost when the production sequence is altered; the cost of a skipped or reversed sequence is normally higher than maintaining the regular sequence. The production scheduling has to be carefully concerned with reducing unnecessary changeover from period to period.

The Minimum lot size of the production is another characteristic of continuous production. The minimum size of production for each product must be produced before changeover to other product. According to this limitation, the inventory helps to minimize the production cost by carrying the product that exceeds the current demand to the next period.

Adding lot-size and sequence consideration into production scheduling leads to a transformation of the calculation from the Linear Programming (LP) to the Mixed Integer Programming (MILP), caused by lot-size and production sequence consideration. The setting up status in each period are defined as the binary variables, in addition, the min-lot consideration are casted as the integer variable. Both sets of the discrete variables add complexity into the formulation. The General Lotsizing and Scheduling Problem (GLSP), classified as the Non-Deterministic Polynomial-time hard Problem (NP-hard), requires a huge number of computational time for solving this problem. The exact methodology to solve this problem will be facing the fractional set of the binary variables during computation. These fractional parts are the cause of the weak bound in the branch-and-bound technique that influences the branching technique in an appropriate direction and the results of the large number of iterations in the computation. Enormous resources such as memory and computational time are required for finding the solution.

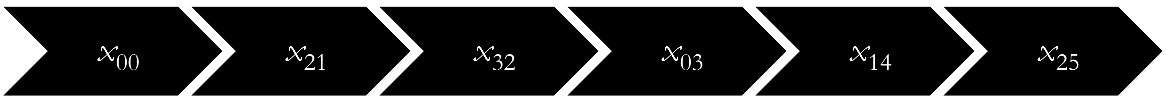

Fig. 1. Sample of the production schedule.

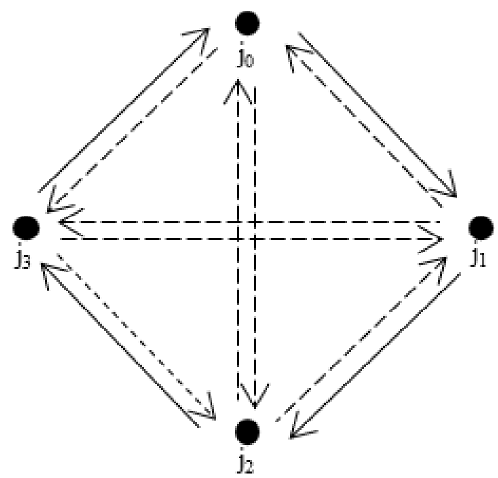

Fig. 2. Changeover Cost from product i to product j (s_{ij}) .

The example of the Production Schedule is shown in Fig. 1. The production number in each period is indicated as x_{js}. In the micro-period s_0 the planned production was on product j_0 with the amount of x_{00}. In these 6 example periods the production was on product j_0, j_2 ,j_3 ,j_0 ,j_1 and j_2 respectively. The plan shown that change over from product i to product j was orderly assigned. The Changeover Cost from product i to product j (s_{ij}) is illustrated in Fig. 2. The solid line shows the minimum changeover cost from one product to another product and the dash line shows the more expensive changeover from product to product.

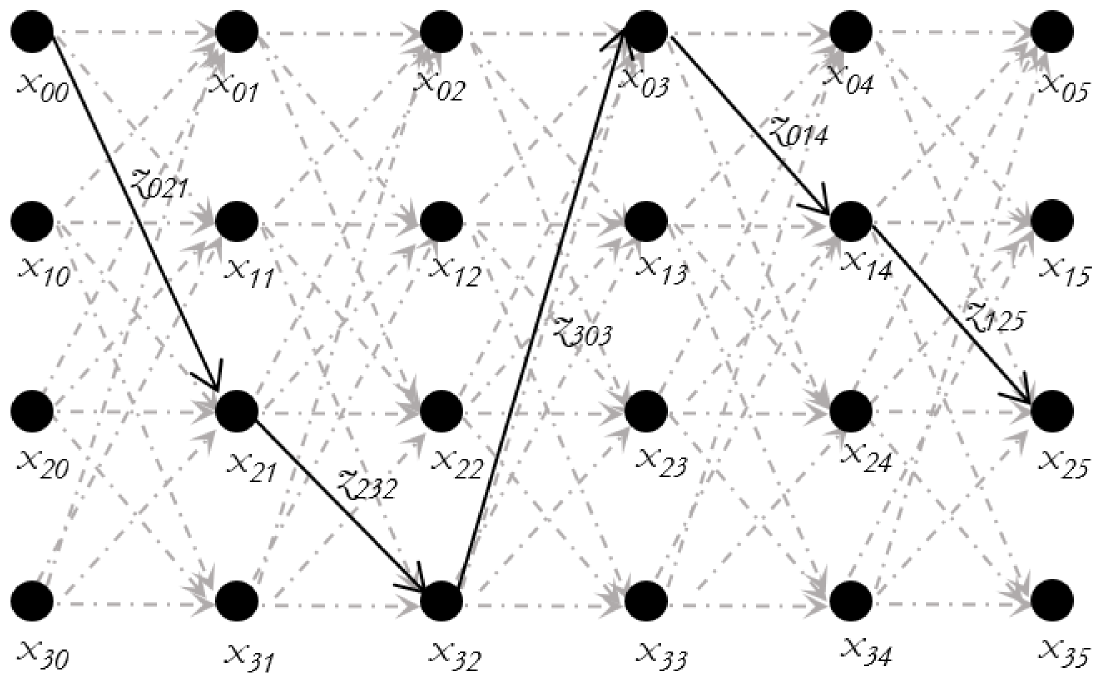

Fig. 3. Changeover Variable from product i to product j in time s (z_{ijs}).

The large number of binary variable is the Changeover variable (z_{ijs}) and this binary variable indicates the changeover stage from product i to product j in time s. The number of z_{ijs} is equal to the number of the product multiplied by the number of product and multiplied by the number of the micro-period ($|J|^2 \times |S|$). This set of variables will increase exponentially when the number of products is increased. As a result, the increasing of this variable is the adding complexity of the problem. During the LP relaxation, this set of variables will be the fractional part, the part that makes weak bound on the problem.

This paper proposes an improvement of the GLSP by formulating a model tackling the important part of the formulation and the bound from LP relaxation that is used for determining the solution gap of the incumbent solution. The tighter bound may lead to the calculation of the exact methodology to effectively answer this sophisticated problem with less memory usage and computational time.

2. Related Works/ Literature Review

2.1. Formulation

Fleischmann and Meyr (1997) [1] proposed a complex version of the production scheduling problem known as the *General Lotsizing and Scheduling Problem* (GLSP), with sequence considerations added into the formulation. In each setup, changing from one product to other products affects the production cost as it also depends on the sequence of the changeover. With the sequence consideration included, the discrete variables were introduced into the formulation by defining and setting up the variables between the changed products.

The GLSP contains an objective function that consists of two parts. The first part is an inventory holding cost of each Product in each Macro-Period. The second part is a setup cost of production changeover from Product *i* to Product *j* in each Micro-Period (if needed). This is subject to the constraints to cover the demand volume of each Product to be fulfilled in each Macro-Period with the number of inventory carried to the next Macro-Period. The capacity constraints cover how much machine time in each Macro-Period and how much machine time is used for production. This constraint calculates how much machine time is used to produce each Product in each Macro-Period, and how much machine time is used in the production setup to transform from Product *i* to Product *j*. The other set of constraints is used to determine which Product has been set up for Product *j* in Micro-Period *s*, to set each production lot needed to produce at least a minimum run for each product and to set up for only one Product in each Micro-Period. The last set of constraints is used to determine when the production changeover from Product *i* to Product *j* occurs. With GLSP the number of discrete variables required is huge, and this formulation does not cover backlogging.

2.2. Solution Methodology

Production scheduling has been categorized into 5 groups by Drexl and Kimms (1997) [2]: 1) the capacitated lot sizing problem, 2) the discrete lot sizing and scheduling problem, 3) the continuous setup lot sizing problem, 4) the proportional lot sizing and scheduling problem, and 5) the general lot sizing and scheduling problem. The *Capacitated Lotsizing Problem with sequence dependent Setup Cost* (CLSD) was introduced by Haase (1996) [3] and is similar to The *General Lotsizing and Scheduling Problem* (GLSP) Fleischmann and Meyr (1997) [1]. The GLSP has been used in many recent studies in which researchers have introduced improvements in both the exact and heuristics approach.

The Lagrangian Relaxation method was used by Chen and Thizy (1990) [4] on several constraints such as setup, demand and capacity constraints. The method is also involved with the subgradient optimization and column generations in node-arc formations using the shortest path technique. The lower bound improvement, achieved by adding cutting plane to the formulation, was introduced by Belvaux and Wholsey (2001) [5]. It categorized the startup and changeover into four parts: 1) Small bucket model, 2) one setup per period, 3) two setup per period, and 4) big bucket model with changeovers. In addition, they also introduced the minimum production runs and full-capacity production. However, the drawback of this formulation is the changeover variable that uses a discrete variable that makes the model more complex. An improvement of exact methodology was the modified branch and bound enumeration method, which was introduced by Haase and Kimms (2000) [6]. It stated that in period *T*, perform a branching step by choosing a sequence and doing calculations to choose whether the model needs to move on to period one step by step by doing backtracking in-between if necessary. The multi-level MILP formulation for Medium-Range Production Scheduling of a Multiproduct Batch Plant was introduced by Lin et al. (2002) [7] by dividing the problem into the category stage and product stage. This approach applied the lower memory space usage to lower bounding in iterations.

There are many techniques to approaching the NP-Hard problem. In the survey of Woeginger (2003) [8] it was shown that the researcher used Dynamic Programming, Pruning the Search Tree, Preprocessing the Data and Local Search depending on the characteristics of the problem. The Mixed Integer Dynamic

Optimization (MIDO) was used by researchers such as Held and Karp (1962) [9], Bansal et al. (2003) [10], Prata et al. (2008) [11], and Chu and You (2013) [12]. The Mixed-Integer Linear Fractional Programming (MILFP) also introduced for the cycle process scheduling problem by You (2009) [13]. The Searching over Separator Strategy also was introduced by Hwang et al. (1993) [14] by dividing the problem into two subproblems in which the results from both subproblems were combined as an optimal solution. Furthermore, Drori and Peleg (2001) [15] proposed an algorithm recursively partitioned the problem domain and eliminated some branches during calculations. All techniques have been used for tackling the optimization of the complex and time consumed problem.

The heuristics methodology was used by various researchers. Meyr (2000, 2002) [16, 17] also improved his methodology by using the dual network flow to re-optimize the sub-problem. This methodology evaluated the new candidate added back to the current solution to find the better solution using dual price. This methodology also used in both single machine consideration and the multi machines scheduling. The three steps of heuristics were published by Gupta and Magnusson (2005) [18] by dividing them into three steps: Initialize, Sequence and Improve. The Initialize step is used to find initial solutions by determining production quantities without sequence consideration. The Sequence step consists of finding the least-costly production within each period. The last step, the Improve step is to refine production quantities and production sequence in regards to decreasing the total costs. The hybrid of the mathematical programming and the local search methods were published by De Araujo et al. (2007) [19]. This hybrid method is called the relax-and-fix methodology. It divides the problem into two levels: 1) solving some relaxed integer variables and solving relaxed problems, and 2) re-specifying some integer variables and then solving partially fixed problems. The heuristics methodology is used to solve both steps to find feasible solutions. The dynamic programming and heuristics technique that focus on binary variables related to sequences was introduced by Kovács et al. (2009) [20] while running a pre-processer to determine the items that should appear in an optimal solution. Combining simulation and optimization was also introduced by Kämpf and Köchel (2006) [21]. The simulation was used to find the optimal parameters before feedbacking to the optimization for optimizing and assessing the value from the simulator for the possibility of optimality.

3. Methods

The *General Lotsizing and Scheduling Problem using Two Phases with External Supply* (GLSP-TE) tackles the computational time consumption of the GLSP by separating the computation steps into two phases. Phase One is the Pattern generation with a Specific Batch size and Capacity Phase Two is the Production Allocation using a Specified Pattern. An additional feature of the GLSP -TE over the GLSP is introducing the external supply to adding up the other supply covering the limited capacity.

Phase One performs the approximate optimization to find the production pattern as shown in Eq. (1) – (9), using the following notation to formulate problem:

Set:

S_t	: Set of Micro-periods s belonging to Macro-period t
J	: Set of Products
T	: Set of Macro-Period
S	: Set of Micro-Period

Parameters:

K_s^*	: Modified Capacity (time) available in Micro-period s
a_j	: Capacity consumption (time) needed to produce one unit of j
h_j	: Holding costs of Product j (per unit and per Macro-Period)
C_{jt}	: External supply unit cost of Product j in Macro-period t
s_{ij}	: Setup costs of changeover from Product i to Product j
d_{jt}	: Demand of Product j in Macro-period t (units)
I_{j0}	: Initial inventory of Product j at the beginning of the planning horizon (units)
y_{j0}	: Equal to 1 of the machine is set up for Product j at the beginning of the planning horizon (0 otherwise)

Variables:

$I_{jt} \geq 0$	: Inventory of Product j at the Macro-period t (units)
$W_{jt} \geq 0$	: Number of external supply of Product j in Macro-period t (units)
$x_{js} \geq 0$	: Quantity of Item j produced in Micro-period s (units)
$y_{js} \in \{0,1\}$	: Setup State: $y_{js} = 1$, if the machine is setup for Product j in Micro-period s (0 otherwise)
$z_{ijs} \in \{0,1\}$	: Take on 1, if a changeover from Product i to Product j take place at the beginning of Micro-period s (units)

Phase One: Pattern generation with Specific Batch size and Capacity

$$MIN \sum_{jt} h_j I_{jt} + \sum_{ijs} s_{ij} z_{ijs} + \sum_{jt} C_{jt} W_{jt} \tag{1}$$

Subject to:

$$I_{jt} = I_{j,t-1} + \sum_{s \in S_t} \frac{K_s^*}{a_j} y_{js} + W_{jt} - d_{jt} \qquad \forall t \in T,\ \forall j \in J \tag{2}$$

$$\sum_j y_{js} = 1 \qquad \forall s \in S \tag{3}$$

$$z_{ijs} \geq y_{i,s-1} + y_{j,s} - 1 \qquad \forall s \in S,\ \forall i \in I, \forall j \in J \tag{4}$$

$$I_{jt} \geq 0 \qquad \forall t \in T,\ \forall j \in J \tag{5}$$

$$W_{jt} \geq 0 \qquad \forall t \in T,\ \forall j \in J \tag{6}$$

$$x_{js} \geq 0 \qquad \forall t \in T,\ \forall j \in J \tag{7}$$

$$y_{js} \in \{0,1\} \qquad \forall s \in S,\ \forall j \in J \tag{8}$$

$$z_{ijs} \in \{0,1\} \qquad \forall s \in S,\ \forall i \in I, \forall j \in J \tag{9}$$

The Objective Function (Eq. (1)) consists of three costs including inventory carrying costs of Product j and the cost of the external supply for Product j in Macro-period t and setup costs of change over from Product i to Product j in Micro-period s. The formulation is subjected to 3 sets of constraints. The first set of constraints represents the conservation of flow that determines the inventory of each Product j and the external supply needed, which satisfies demand of Product j in Macro-period t (Eq. (2)). The K_s^* is the pre-defined capacity in each Micro-period that is greater than the actual capacity in the Micro-period calculated from Macro-period capacity to represent the minimum lot. The second and third set of constraints are responsible for the Setup State from Product i to Product j (Eq. (3) and Eq. (4)). The other constraints are the Non-Negativity constraint on variable I_{jt}, W_{jt}, x_{js} (Eq. (5) to (7)) and the Binary constraint on variable y_{js}, z_{ijs} (Eq. (8) and Eq. (9)).

After Phase One calculations, the Setup State variables (y_{js}) have been passed to Phase Two to be used as Setup State to calculate the amount of Production Units, as provided in formulations (Eq.10) - (Eq.19) using the following notation to formulate the problem:

Set:

S_t	: Set of Micro-periods s belonging to Macro-period t
J	: Set of Products
T	: Set of Macro-Period
S	: Set of Micro-Period

Parameters:

K_t	: Capacity (time) available in Macro-period t
a_j	: Capacity consumption (time) needed to produce one unit of j
h_j	: Holding costs of Product j (per unit and per Macro-period)
C_{jt}	: Unit Cost of External supply for Product j in Macro-period t
s_{ij}	: Setup Costs of Changeover from Product i to Product j
d_{jt}	: Demand of Product j in Macro-period t (units)
m_j	: Minimum lotsize for Product j

I_{j0} : Initial inventory of Product j at the beginning of the planning horizon (units)

y_{j0} : Equal to 1 of the machine is set up for Product j at the beginning of the planning horizon (0 otherwise)

Variables:

$I_{jt} \geq 0$: Inventory of Product j at the end of the planning horizon (units)

$W_{jt} \geq 0$: Number of External supply of Product j in Macro-period t (units)

$x_{js} \geq 0$: Quantity of Item j Produced in Micro-period s (units)

$y_{js} \in \{0,1\}$: Setup State: $y_{js}=1$, if the machine is setup for Product j in micro - period s (0 otherwise)

$z_{ijs} \in \{0,1\}$: Take on 1, if a changeover from Product i to Product j take place at the beginning of Micro-Period s (units)

Phase Two: Production Allocation using Specified Pattern

$$MIN \sum_{jt} h_j I_{jt} + \sum_{ijs} s_{ij} z_{ijs} + \sum_{jt} C_{jt} W_{jt} \quad (10)$$

Subject to:

$$I_{jt}=I_{j,t-1}+\sum_{s\in S_t} x_{js}+W_{jt}-d_{jt} \quad \forall t\in T,\ \forall j\in J \quad (11)$$

$$\sum_{j,s\in S_t} a_j x_{js} \leq K_t \quad \forall t\in T \quad (12)$$

$$z_{ijs} \geq y_{i,s-1}+y_{j,s}-1 \quad \forall s\in S,\ \forall i\in I, \forall j\in J \quad (13)$$

$$x_{js} \geq m_j(y_{js}-y_{j,s-1}) \quad \forall s\in S,\ \forall j\in J \quad (14)$$

$$I_{jt} \geq 0 \quad \forall t\in T,\ \forall j\in J \quad (15)$$

$$W_{jt} \geq 0 \quad \forall t\in T,\ \forall j\in J \quad (16)$$

$$x_{js} \geq 0 \quad \forall t\in T,\ \forall j\in J \quad (17)$$

$$y_{js} \in \{0,1\} \quad \forall s\in S,\ \forall j\in J \quad (18)$$

$$z_{ijs} \in \{0,1\} \quad \forall s\in S,\ \forall i\in I, \forall j\in J \quad (19)$$

The Objective function in (Eq. (10)) considers the inventory carrying cost and the cost of the external supply for Product j in each Macro-Period t, and also the setup cost for changing Product i to Product j in Micro-Period s. The sets of constraints cover demand satisfaction on (Eq. (11)). Capacity consideration takes place on (Eq. (12)), whereas the switching cost is considered on (Eq. (13)). The minimum lot-size is considered on (Eq. (14)) by the pattern from Phase One. The other constraints are the Non-Negativity constraint on variable I_{jt}, W_{jt}, x_{js} (Eq. (15) to Eq. (17)) and the Binary constraint on variable y_{js}, z_{ijs} (Eq. (18) and Eq. (19)).

4. Computational Test

The formulation testing is divided into two parts, model validity and model solvability. The model validity testing uses a reduced size of problem and the model solvability is tested on various scenarios in a larger size of problem. All tests are performed on an IBM compatible PC with Intel i7 3770 processor, which has 4 cores, 8 threads and 16GB RAM as hardware. The software used in this test is IBM ILOG CPLEX 64 bit version 12.4 with IBM ILOG Concert technology interface with Microsoft Visual Studio C#.NET 2012 using .NET Framework 4.0.

4.1. Model Validity

The testing of model validity performed on a reduced size of problem to prove that the GLSP -TE can provide the optimality up to GLSP by reducing the commodity to just three commodities, three Macro-

Periods with a total of nine Micro-Periods. The test found that GLSP -TE can perform the same result of the objective value.

4.2. Solvability

4.2.1. Test scenarios

The proposed formulation was tested by 8 scenarios with adjusting $K_S{}^*$ parameters to 22 values to evaluate the formulation. In this paper, the $K_S{}^*$ will be tested as a set of data from min-lot to more than two times of the min-lot itself. The scenario lists are shown in Table 1.

Table 1. Scenario list.

Scenario Code	Demand Continuity	Product Variety	Demand Fluctuation	Min-Lot Violation	Total Demand Level
SCN1	Continuous	No	Low	No	Under Capacity
SCN2	Disjointed	Missing middle demand/ Skipped period	Low	No	Under Capacity
SCN3	Disjointed	One commodity in almost period	Low	No	Under Capacity
SCN4	Interval Demand commodity	All commodity in some period	Moderate	No	Under Capacity
SCN5	Continuous	No	High	No	Possible to over capacity
SCN6	Continuous	No	Moderate	No	Possible to over capacity
SCN7	Continuous	No	High	Yes	Possible to over capacity
SCN8	Continuous	No	Very High	Yes	Possible to over capacity

In each scenario, the computation performed three solution gaps (5%, 10%, and 20%) to determine the computation time and solvability of the model. The timeout was set to 120 minutes to determine the applicability of the model in practice.

4.2.2. Computation results

The overall results in all solution gaps can be divided into 2 categories. First, the solution that can be found within the solution gap at 86% of all runs. Second, the solution that cannot be found within the solution gap in the defined amount of time but a feasible solution was made at 14% of all runs. From all tested scenarios, 100%, 91.48%, and 66.48% of optimal solutions are obtained in the solution gaps of 20%, 10%, and 5%, respectively. The feasible solution with an average gap of about 8.8% is provided in all test runs, implying that the formulation performs a tighter bound.

The calculation results from all scenarios show that the distribution of computational time is related to the solution gaps. With the wide gap (20%), the model can be solved fastest; whereas, for the middle gap (10%) all the tests can be solved within 30 minutes. The smallest gap (5%) shows various computational time, where some tests reached the timeout that is defined in the parameters.

The distribution of iteration numbers is shown in Fig 4. The computational time and the number of iterations on the tests depend on the solutions gap tends to be in enormous number with much more iterations, which means that the solver did huge a number of iterations trying to close the gap but could not find the better solution within the defined timeout. In SCN7 and SCN8 the computational time and the number of iterations appeared in small numbers and fewer runs in the indicated timeout. For SCN1, SCN2 and SCN3 the number of iterations were spread higher than the first group with more runs marked in the defined timeout as well. The SCN4, SCN5 and SCN6 is the group that the number of iterations are closed to

SCN7 and SCN8 but the computational time spread up to the defined timeout, which means that the solver tried to close the solution gap but could not find the solution within the gap.

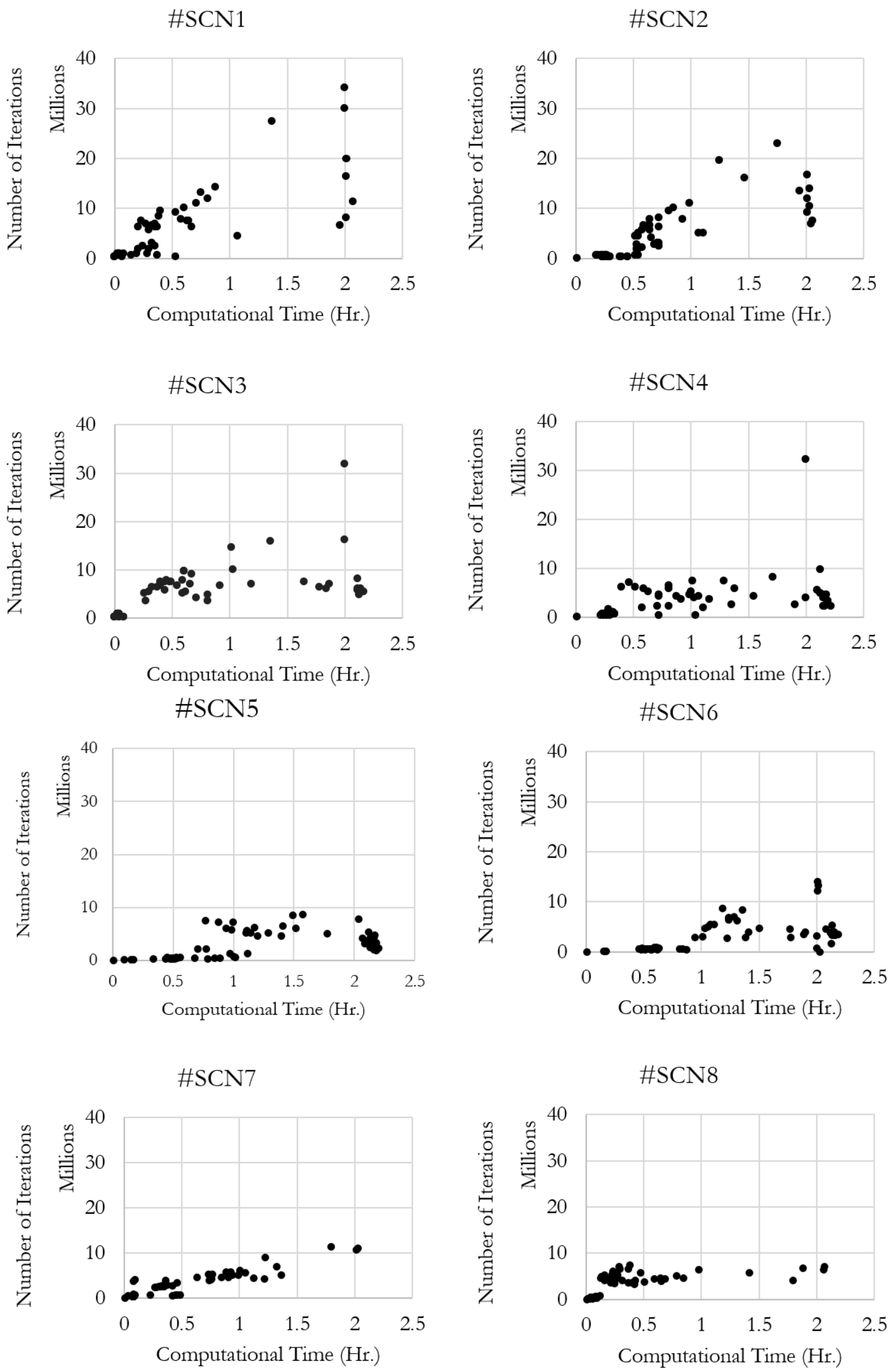

Fig. 4. Computational time and number of iterations for all scenario runs.

The test results are ranked into 3 groups according to the value of the third quantile, the value of the first quantile and the median, which can be categorized into the best result, the good result and the moderate result.

The best result is performed on SCN7 and SCN8, with high and very high demand fluctuation and Min-lot violation. Phase One performs the best computational time to reach the gap with a tighter bound, from the pre-defined production volume in each Micro-period, as shown in Fig. 5, Fig. 6, and Fig. 7.

The good result is performed on SCN1, SCN2 and SCN3. The median value and first quantile of the good result is lower than the third group as shown in Fig. 5, Fig. 6, and Fig. 7. The characteristics of this groups is the low demand fluctuation with some skipped demand spreading in all period (SCN2 and SCN3)

The moderate result is shown in SCN4, SCN5, and SCN6. The missing demand is an interval for all commodities in the same period. SCN4 shows a moderate demand fluctuation whereas SCN 5 and SCN6 show continuous demand with high and moderate demand fluctuation. Phase One requires a long computational time for both the median and third quantile, as shown in Fig. 5, Fig. 6, and Fig. 7.

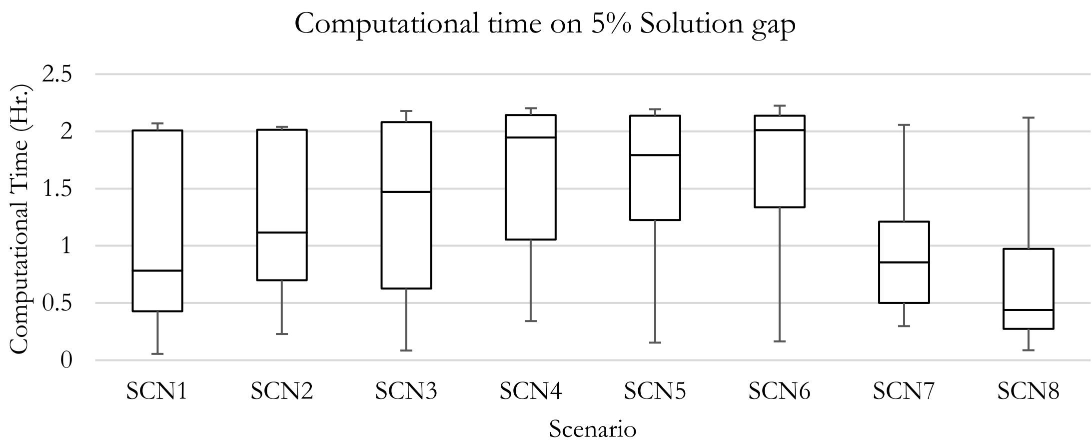

Fig. 5. Computational time on each scenario using 5% solution gap.

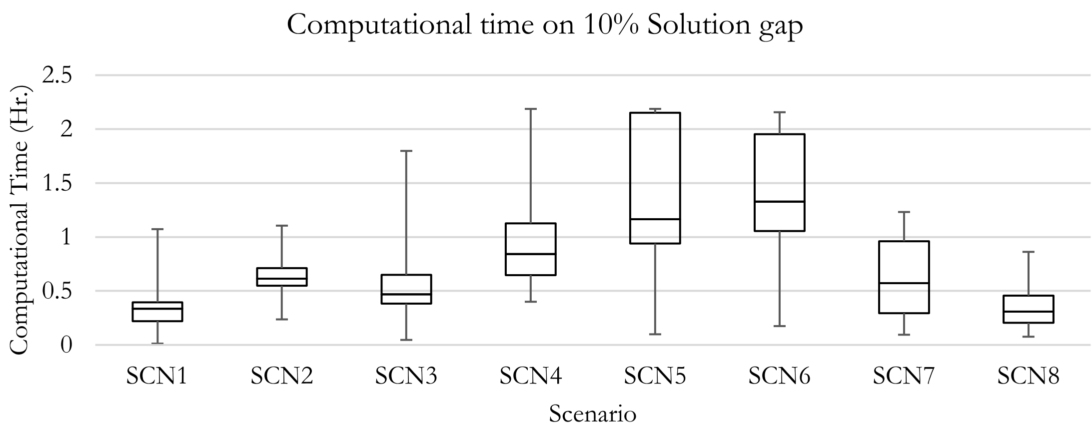

Fig. 6. Computational time on each scenario using 10% solution gap.

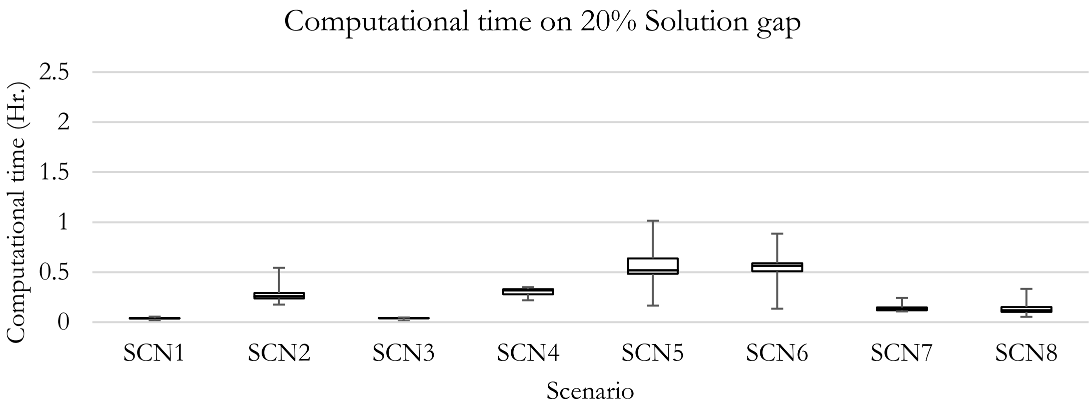

Fig. 7. Computational time on each scenario using 20% solution gap.

The stress test of GLSP-TE is performed on scenarios that performed the best and worst scenarios from previous tests, the sixth scenario (SCN6) and the eighth scenario (SCN8). The tests are performed in 2 sets, first, a scaling product from 4 products to 30 products, and second, a scaling number of a micro-period from 4 to 24 micro-period in each macro-period in scenarios that have 4 products. Both sets used a 2 hours timeout computation and 5% solution gap compared with GLSP with the same parameters except K_s^* in GLSP-TE that will try 10 parameters and use the best results to compare with GLSP. The first set shows that all tests on GLSP were timeout with a solution gap of more than 99%. The GLSP-TE can perform optimality within the solution gap for most runs for SCN6 and all runs for SCN8, as shown in Fig. 8 and Fig. 9. The second set shows that the GLSP perform optimality with the solution gap in a scenario that has a binary variable of less than 10,000 variables. In SCN8 for scenarios that have more binary variables the optimizer report timed out with a feasible solution and did not achieve defined solution gap. In SCN6 the solver could not find an optimal solution since the number of the product was 4, but the GLSP-TE could perform all runs in the optimal stage, as shown in Fig. 10 and Fig. 11. The result showed that GLSP-TE has a tighter bound than GLSP from dividing the formulation into 2 phases.

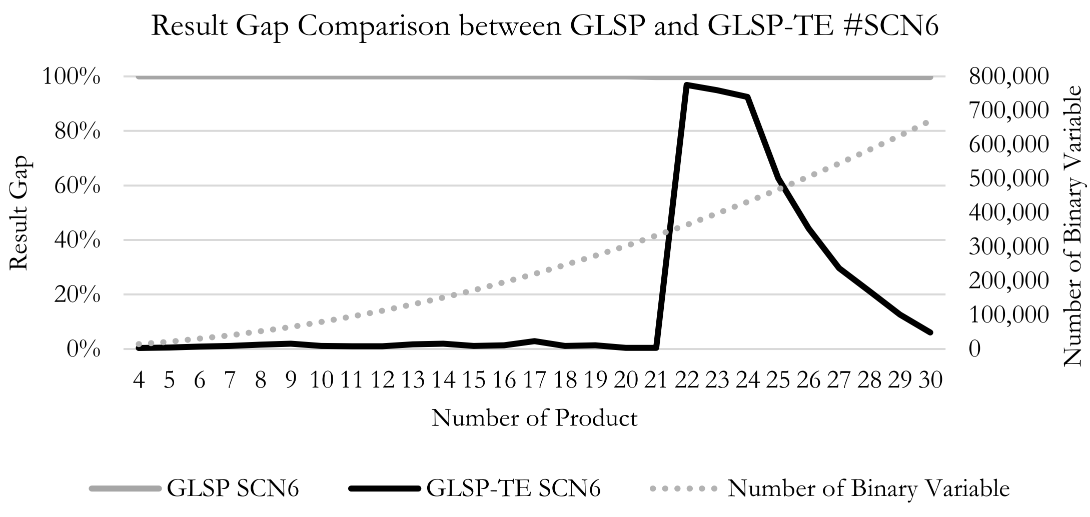

Fig. 8. Result Gap Comparison between GLSP and GLSP-TE on SCN6.

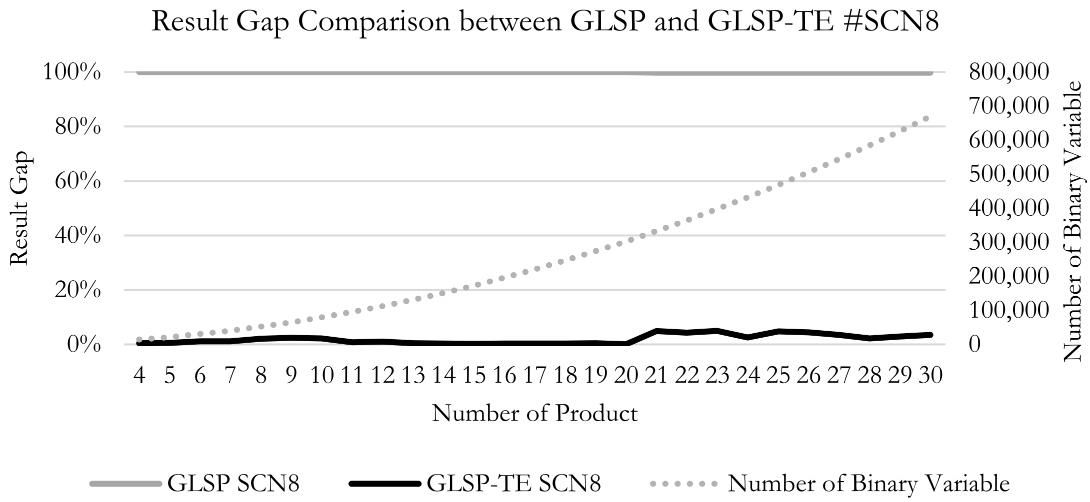

Fig. 9. Result Gap Comparison between GLSP and GLSP-TE on SCN8.

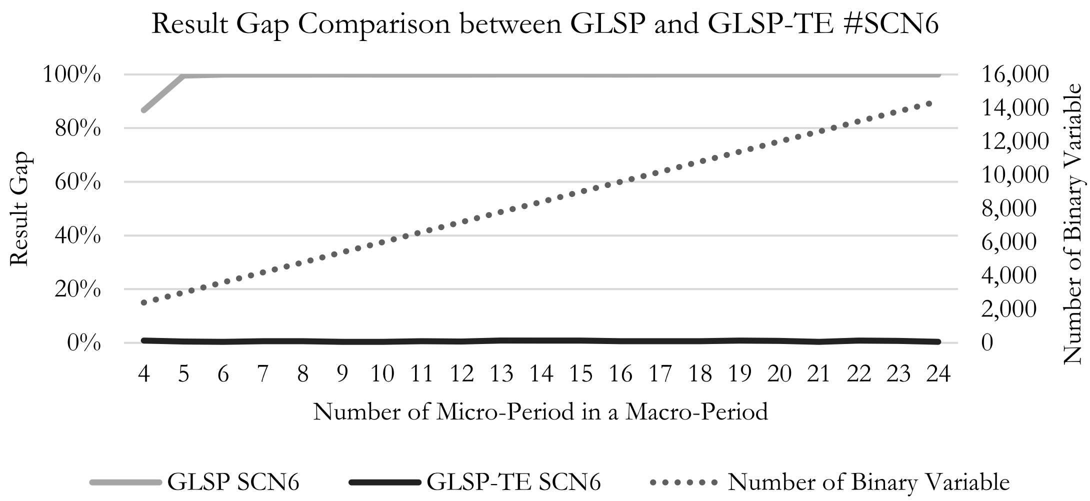

Fig. 10. Result Gap Comparison between GLSP and GLSP-TE on SCN6.

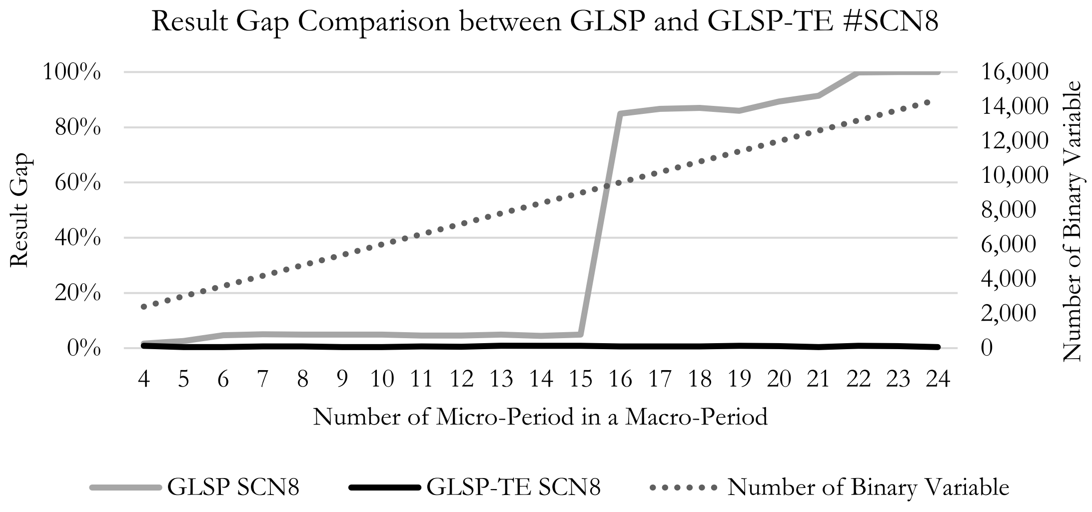

Fig. 11. Result Gap Comparison between GLSP and GLSP-TE on SCN8.

4.3. Model Result

The improvement GLSP -TE has been tested by running the optimization with the same scenarios at 5% of the solution gap and with a 120 minutes timeout on the same machine configuration and environment. Phase One was tested by sampling the $K_S{}^*$ parameters into 22 values and using the average of the objective function value of Phase Two in each scenario compared with the objective function value of the GLSP. Most of the runs using GLSP do not reach optimality within defined time but a feasible solution can be obtained, except for SCN5, which reached an optimal solution within the 5% gap. In addition, only SCN6 did not have an improvement of the objective function value. The objective value of SCN1, SCN2, SCN3, SCN4, SCN5, SCN7, and SCN8 from the tighter bound of the GLSP -TE were improved to 40%, 36%, 43%, 15%, 5%, 69%, and 72%, respectively, as shown in Fig. 12.

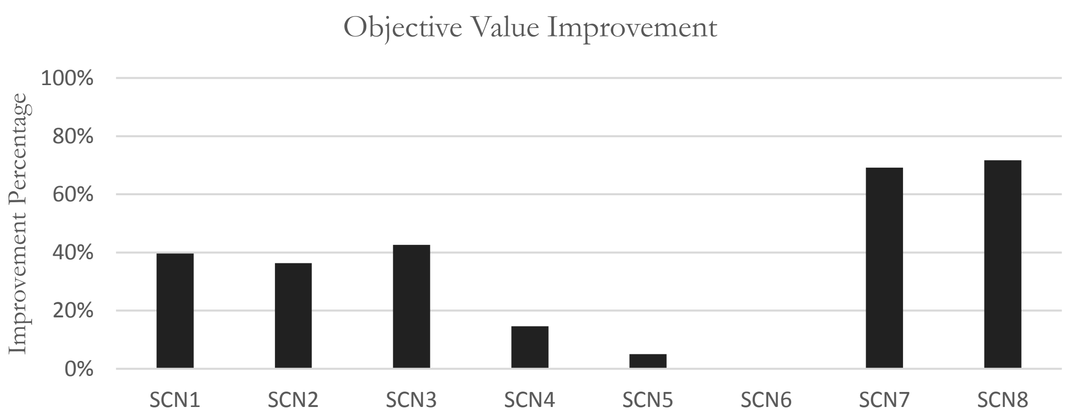

Fig. 12. Percentage of Proposed formulation objective improvement.

The computational time and improvement of the objective function value shown that the GLSP -TE performed well in scenario with has high level of the branching in both demand fluctuation and min-lot violation. Less computational time and high level of the objective function value improvement are obtained from this GLSP-TE. The moderate performed is on the scenario that has low demand fluctuation by the median of the computational time and the objective function value improvement.

5. Discussion

The GLSP -TE is based on separating the most fractional part into Phase One, the pattern generation, which comes with a pre-defined lot-size for each commodity to perform an approximate optimization in calculating the production pattern. Depending on the number of the pre-defined lot-size, the results of the pattern generation might remove some feasible solution, causing the infeasible solution in the following step. From the reasons stated, the testing of the formulation is performed with multiple pre-defined lot-sizes in each scenario and gap, to ensure that the result will reach a feasible solution. All the results show at least feasible solutions without any single infeasible solution. The setup pattern from Phase One is passed to Phase Two, to calculate the final production volume with consideration for demand, the inventory carrying cost, and capacity to formulate the solution.

The quality of the result has been tested by reducing the problem size due to the limitation of running by GLSP. The result in the reduced problem returns the same optimality for both formulations. In the large size problem, Phase One and Phase Two perform less computational times with a better objective function value compared with GLSP in most tested scenarios.

From all tests performed, it can be concluded that the GLSP-TE provides optimality level with appropriate computational time. The future aspect of this formulation can be continued on the pre-defined lot-size calculation that can be improved for the precision and avoiding the infeasible effect in the next phase. Moreover, the gap in the multiple production line has to be considered for further studies.

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