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Article

Nonlinear Finite Element Analysis to Evaluate Lateral Torsional Buckling Moment of Elliptical Cellular Steel Beams

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Abstract. In evaluation the lateral torsional buckling moment of Elliptical Cellular beams, it cannot be accurately calculated using a simple model because the sectional properties of the Elliptical Cellular beam are not constant along the beam length. Consideration of sectional properties at the web-post section will certainly lead to unconservative results, while consideration of sectional properties at the web-opening section is also unreal and yield conservative design. Therefore, the main objective of this paper is to present the guideline to evaluate the lateral torsional buckling moment capacity of Elliptical Cellular beam for conservative and more precise results. The study has been performed throughout finite element analysis in both elastic and inelastic buckling behavior subjected to equal beam-end moments. More than 15 sections, 120 of beam models were simulated and analyzed with verified finite element modelling by using shell elements. The results of this study show that calculation the lateral torsional buckling moment of Elliptical Cellular beam with the original equation given in AISC standard using sectional properties at the web-post section gives unsafe results with approximately 5% error in elastic range and up to 20% error in inelastic and plastic range. Nonetheless, evaluation the lateral torsional buckling moment by using the proposed reduction factor has yielded the conservative results, yet more precision, more economic and less distribution of the results than the evaluation using sectional properties at the web-opening section.

Keywords: Elliptical cellular beam, lateral torsional buckling, finite element analysis.

1. Introduction

Cellular beam is a circular web-openings steel beam fabricated by cutting the web of the standard hot-rolled wide flange section with certain pattern into two pieces and then reassembling the two halves again by welding. As a result, the beam depth is increased but leaving the circular holes at the beam web. The main engineering advantage is that the stiffness and the strength can be increased with the same amount of material. Another web-openings beam is also so-called by the configuration of opening shape, such as Castellated beam for hexagonal web-openings shape, Angelina beam for sinusoidal web-openings shape, and presented in this paper, Elliptical Cellular beam for elliptical web-openings shape. This modern Elliptical Cellular beam has the elliptical web-opening configuration that is developed base on the typical hexagonal shape from design of Castellated beam by BS 5950 and BS 449 standard [1] (see Fig. 1). The web-opening shape has to use technology for cutting and welding in elliptical curves pattern, but little more effective advantages than Castellated beam and Cellular beam as previous research [2-7] that can be drawn as followings:

- The elliptical web-opening shape has less critical opening length than circular web-opening shape, which affect to demote occurring of Vierendeel mechanism failure.
- The curvature of elliptical opening effectively contribute to reduce stress concentration at corners of hexagonal opening shape.
- The curvature of elliptical opening that take place to corner of hexagonal opening has reduce tearing and fracture mechanism.
- The Elliptical Cellular beam developed base on typical hexagonal web-opening shape has fix on dimension ratio, suitable for practical application.
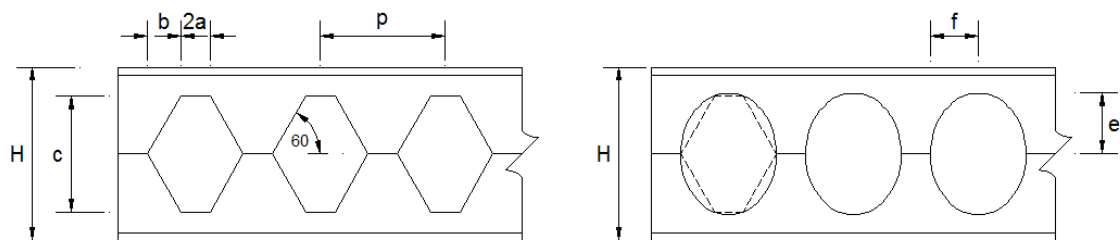

Fig. 1. The configuration of elliptical web-opening developed base on typical hexagonal web-opening shape from the design of Castellated beam with BS 5950 and BS 449 standard [18].

There are many researches on behavior and failure modes of web opening beams that the beam length is not long [8-12]. But, in long span length, fabricating original steel beam into Elliptical Cellular beam has a good solution to control the deflection of beam. Subsequently, the beam with long span length will be vulnerable to lateral torsional buckling behavior if no sufficient bracings against lateral displacement of the compression flange or bracings against twist of the cross section. Therefore, to account for the lateral torsional buckling moment of original I-shaped steel beam, general provision of AISC 360-10 [13] specifies the equation to evaluate the lateral torsional buckling moment as Eq. (1) for the range of elastic lateral torsional buckling and Eq. (2) for the range of inelastic lateral torsional buckling. In the equations, C_b is the lateral torsional buckling modification factor for nonuniform moment diagrams, S_x is the elastic section modulus taken about the x-axis, M_p isthe plastic moment, L_b is the length between lateral supports, L_p is the limiting length between plastic and inelastic behavior, L_r is the limiting length between inelastic and elastic behavior, G is the shear modulus of elasticity, and E is the Young's modulus of elasticity.

$$M_{cr} = C_b \frac{\pi}{L_b} \sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w} \tag{1}$$

$$M_n = C_b \left[M_p - \left(M_p - 0.7F_y S_x \right) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \tag{2}$$

Nevertheless, these two equations are derivedfor original I-shape steel beam with constant sectional properties. Thus, it cannot be used to precisely evaluate the Elliptical Cellular beam since the beam has web-openings that make sectional properties, moment of inertia about minor axis I_y, torsional rigidity J and warping constant C_w, vary along the beam length. And the use of sectional properties at the web-opening section to evaluate the lateral torsional buckling moment of the beam is unreal and too conservative. Although there are some previous researches on lateral torsional buckling behavior [14-20] and design [21-23] of Cellular and Castellated beam, these researches have not obviously included all the design ranges of the beams and not extended to the elliptical opening shape. So the purpose of this paper is to conduct the parametric study and propose a simplified coefficient function for precisely evaluate the lateral torsional buckling moment of Elliptical Cellular beam in both elastic and inelastic buckling range throughout finite element analysis.

2. Nonlinear Finite Element Modelling

To investigate the lateral torsional buckling moment capacity of Elliptical Cellular beams in this study, the nonlinear finite element analysis using the commercial software package, ANSYS ver.R14.5 program [24], has been performed with both geometric and material nonlinearities. The assumptions and definitions are described as followings.

2.1. Material Properties

A bi-linear elastic-perfectly plastic stress-strain curve has been adopted to simulate the nonlinear behavior of steel material. The used material parameters are Young's modulus of 2×10^5 MPa, shear modulus of 7.6923×10^4 MPa, tangent modulus of 1×10^{-13} MPa, yield stress of 250 MPa and Poisson's ratio of 0.3. It is worthly noted that the consideration the residual stress of the hot-rolled steel beam and fabricating Elliptical Cellular beam is complicated, yet not essentially found in any literature review, thus, it is not taken into account in the scope of this study.

2.2. Finite Element Type and Mesh

A four-node structural shell element (SHELL181) from ANSYS ver.R14.5 library has been used in this nonlinear finite element analysis to investigate the lateral torsional buckling moment. The element has six degrees of freedom at each node, 3 translations in the x, y and z directions and 3 rotations about the x, y and z axis. The element is suitable for large rotation, large strain nonlinear application and load stiffness effect of distributed pressures. Both full and reduced integration schemes are also supported. The performance of nonlinear buckling finite element convergence shows that the element mesh size of about quarter of flange width (as shown in Fig. 2) is suitable and sufficient to apply in this investigation.
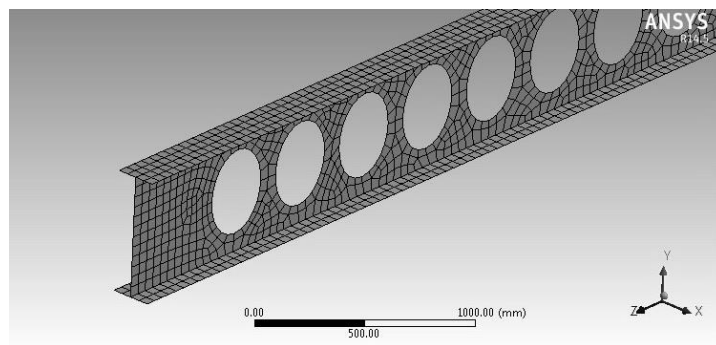

Fig. 2. Shell element mesh size of quarter of flange width (ECB600x200 specimen).

2.3. Boundary Conditions and Load Application

The simple beam with equal end moments has been used in this study. One of the end supports is assigned to be hinge, while another is roller. The rotations about beam's self are restrain at both support, while it is free for in-plane and out-of-plane rotations. Action loads of equal end moments have been obtained as a result by applying the rotation control.

2.4. Initial Geometric Imperfection

In reality, all of the general beams are initial geometrical imperfect due to a manufacturing of hot-rolled steel fabrication process, or additional secondary fabrication process to Elliptical Cellular beams. This initial geometric imperfection can be considered in this nonlinear buckling analysis as initial deformation by using ANSYS program [24]. The procedure of analysis is linked in two parts. Firstly, the initial deformation is obtained from linear buckling by using eigenvalue and eigenvector modal analysis. The obtained initial deformation can be multiplied to enlarge magnitude of the deformation. Then, the finite element model with initial deformation will be updated and import to part of nonlinear buckling analysis. Nevertheless, the shape and location of geometric imperfections in reality are varied and hard to discern. Therefore, to judiciously consider the imperfection of Elliptical Cellular beam in this nonlinear buckling analysis, the initial deformation will be set to half-sine lateral torsional buckling mode which is the first mode shape of linear buckling with the magnitude of 0.0005 of the span length (L/2000).

3. Verification of Nonlinear Finite Element Model

In order to accurately apply the nonlinear finite element analysis to Elliptical Cellular beam in this study, the verification of finite element model has been performed by finite element convergence and comparing two simulated models with two experimental test results, by J. Nseir [21] and K. D. Tsavdaridis [5]. The model compared with J. Nseir was simulated using HEA 340 Cellular beam with the length of 7500 mm. The material properties are referred with a coupon test result with Young's modulus of 230 GPa and the yield stress of 522 MPa. The stress-strain curve of the steel material is modeled as bi-linear elastic-perfectly plastic. The deformation and equivalent stress of the simulated Cellular beam are shown in Fig. 3. Figure 4 shows the result of comparison of load-displacement that represent a good agreement between the experimental test and the nonlinear finite element model. The finite element model shows the ultimate vertical load of 2023 kN, which is merely 2.33% error from the experimental test result.
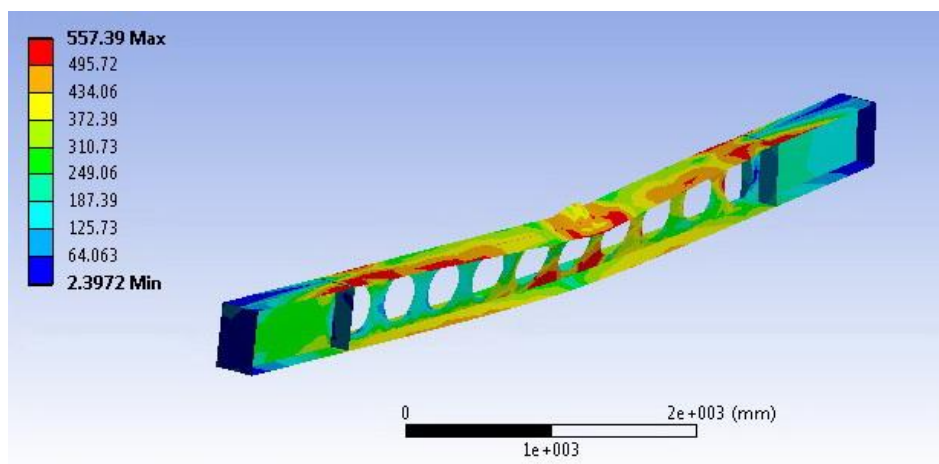

Fig. 3. Equivalent (Von-mises) stress of HEA 340x7500 mm at the maximum loading.
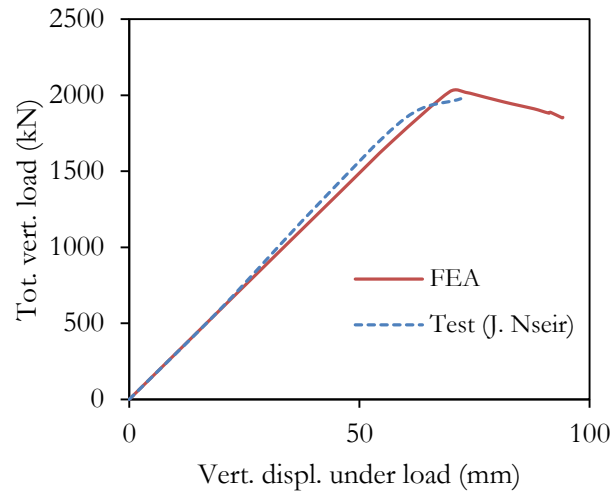
Fig. 4. Comparison of load-deflection of Cellular beam with experimental test [21] (HEA 340x7500 mm).

In the verification comparing with K.D. Tsavdaridis [5], the model was simulated with the specimen B3, simply supported beam with a mid-span concentrated load. This specimen is UB457x152x52 section with vertical elliptical web openings (theta = 10 and R = 0.25d_o). The beam depth h of 449.8 mm, the beam width b of 152.4 mm, the flange thickness t_f of 10.9 mm, the web thickness t_w of 7.6 mm, the d_o of 315 mm, the R of 47.25 mm, the theta of 10, the critical web post width S of 63 mm, the stiffeners at both supports of 10.9 mm, the stiffener at mid-span of 20 mm, and the clear span length L of 1700 mm are all the dimensions used in analysis. The steel material of S355 grade (tensile coupon test with f_y of 375.3 MPa and 359.7 MPa, and $f_{ult.}$ of 492.7 MPa and 480.9 MPa) is simulated using tri-linear material model. The result of the simulation is shown in Fig. 5 to overall compare the equivalent (Von-mises) stress contour and total deformation at the maximum load. The load-deflection comparison has been shown in Fig. 6 using the results of the vertical deflection at mid-span length (dial gauge 3). The results of the load-deflection graph, the stresses, the deformation and the behavior are in good agreement. The failure mode is web-post distortional buckling at the ultimate load of 437.40 kN, 415 kN and 426 kN for this simulated model, K.D. Tsavdaridis's experimental test and Tsavdaridis's finite element analysis, respectively.

(a)   (b)
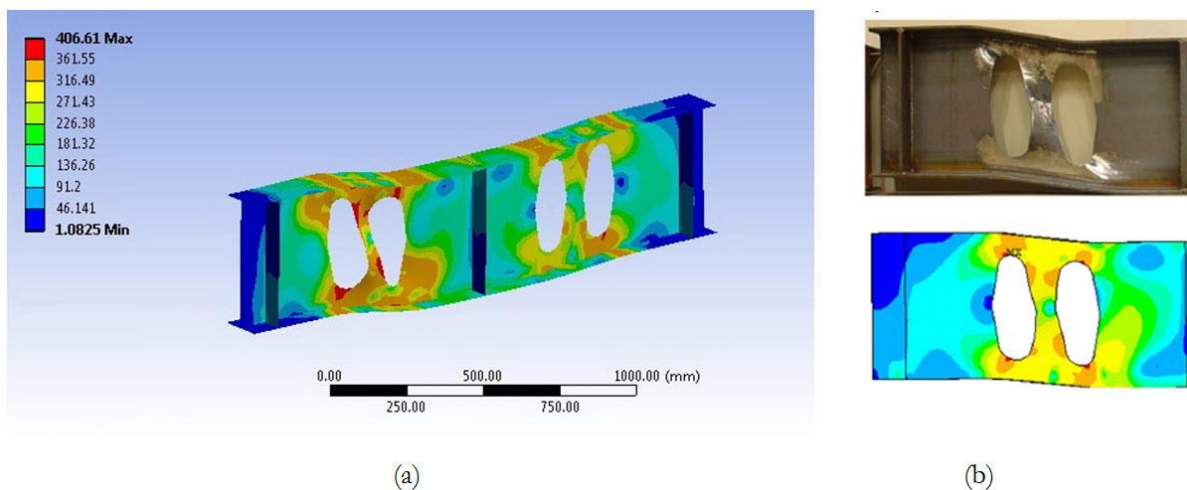
Fig. 5. Comparison of result in specimen B3: (a) simulated model at equivalent (Von-mises) stress along with deformation at last load, (b) result from K.D. Tsavdaridis [5].
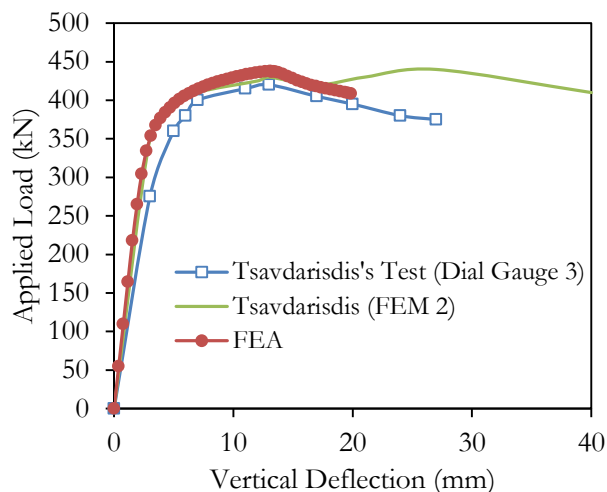
Fig. 6.   Verification of finite element analysis with the experimental test of specimen B3 [5].

4. Parametric Study

After the finite element modelling has been verified carefully, the nonlinear buckling analysis can properly be applied with several cases of Elliptical Cellular beams (ECB) by parametric study that aim to evaluate the lateral torsional buckling moment capacity and to collect the reference results in order to propose a coefficient function for further use in design purpose. The one hundred and twenty models with fifteen sections of Elliptical Cellular steel beam have been extensively chosen for this parametric study. The length of all beam models are provided in both of the elastic and inelastic buckling behavior range, as well as the slenderness of section were chosen in widely various. The grade of material is S250 (grade of 36). Table 1 shows the details of sections and dimensions of the Elliptical Cellular beam. The terminology is defined as followings; H is the height of Elliptical Cellular section, d is the height of the original standard I-beam, B_f is the width of section, t_f is the flange thickness, t_w is the web thickness, e is the half-height of the elliptical opening, f is the half-width of the elliptical opening, and p is the pitch length of the opening. The others Castellation dimensions were followed base on the design of castellated beams for use with BS 5950 and BS 449 standard (Fig. 1). It is noted that, for the all Elliptical Cellular beam models, infill plates were added at both ends of the beam conforming to the BS 5950 standard in order to avoid the concentrated stress and web-post buckling at supports.

Table 1.   Sections and dimensions of Elliptical Cellular beams.

Sections	H (mm)	d (mm)	b_f (mm)	t_f (mm)	t_w (mm)	e (mm)	f (mm)	p (mm)
ECB150x50	150	100	50	7	5	52.47	41.37	108.00
ECB225x75	225	150	75	7	5	78.71	62.15	162.00
ECB297x99	297	198	99	7	4.5	103.89	82.04	213.84
ECB300x100	300	150	100	8	5.5	104.94	82.87	162.00
ECB300x200	300	200	200	12	8	104.94	86.60	216.00
ECB375x125	375	250	125	9	6	131.18	103.58	270.00
ECB450x150	450	300	150	9	6.5	157.41	124.32	324.00
ECB447x149	447	298	149	8	5.5	156.37	123.47	321.84
ECB525x175	525	350	175	11	7	183.65	145.02	378.00
ECB594x199	594	396	199	11	7	207.79	164.08	427.68
ECB600x200	600	400	200	13	8	209.89	165.74	432.00
ECB750x200	750	500	200	16	10	262.36	207.17	540.00
ECB882x300	882	588	300	20	12	308.53	243.63	635.04
ECB900x200	900	600	200	17	11	314.83	248.60	972.00
ECB1350x300	1350	900	300	28	16	472.24	372.90	972.00

Remark: e = half-height of the elliptical opening, f = half-width of the elliptical opening.

5. Result and Discussion

The results of finite element analysis in lateral torsional buckling moment capacity of the Elliptical Cellular beams have been directly compared with the calculation following the equation of AISC 360-10 standard by using the sectional properties at web-post section as seen in Table 2 and Fig. 7. The example results in the table show that the moment capacity calculation following AISC standard is remarkably unconservative. The cause of this unsafe results came from the usages of the only three sectional properties, the moment of inertia about minor axis, I_y, torsional constant, J, and warping constant, C_w.

All of the lateral torsional buckling moment capacity results are plotted to present the relationship between M/M_{FEA} and the modified slenderness, $\sqrt{M_y/M_{ocr}}$, as shows in Fig. 8. The definition of parameters are as followings: M is the lateral torsional buckling moment calculated by AISC standard, M_{FEA} is the moment result from finite element analysis, M_{ocr} is the critical buckling moment, and M_y is the section yielding moment. The result data were widely scatter in the zone of plastic range, inelastic buckling range, and elastic buckling range, which the turning point of elastic buckling behavior to inelastic buckling behavior is about at 1.20 of the modified slenderness. The observation of result data shows that the lateral torsional buckling moment capacity of the beam having the opening at the web is reduced depending mainly on the modified slenderness, which includes the beam length. Furthermore, it is observed that the moment reduction is also depending on the slenderness of the top flange. As noticed in the failure configuration of lateral torsional buckling (Fig. 9), the top flange of the beam undergone the compression until it was buckled and deflected to the lateral direction. Therefore, the guideline to evaluate the lateral torsional buckling moment capacity of Elliptical Cellular beam is properly formulated by using the reduction factor, R_{ECB}, to reduce the moment capacity from the equation given in AISC standard with consideration of sectional properties at the web-post section. This reduction factor, R_{ECB}, has been developed from the curve fitting with linear multiplied regression including two essential variables, the modified slenderness, $\sqrt{M_y/M_{ocr}}$ and the top flange slenderness, $b/2t_f$.

In overall observation of finite element analysis results, it is found that the stiffness of the beams are linear in earliest stage of loading, elastic behavior of beams. But when the onset of lateral torsional buckling occur, one edge of the beam top flange will encounter compressive yielding, and tensile yielding in the other. Subsequently, the load will drop from the maximum point and the beam behavior enters into the inelastic range, while the stiffness of beam was greatly reduced resulting from lateral displacement of the top flange, downward displacement and twist angle of the section.

The example of finite element analysis results on structural beam behavior are plotted in load-deflection graph as shown in Fig. 10 for comparison the plain web and Elliptical Cellular beams. The directional deflections at mid-span are particularly separated into vertical and horizontal direction. The results show that the beams size of ECB150x50x1429 and ECB450x150x4288, which the lengths are in the inelastic buckling range, have displacement at mid-span length moving to the lateral direction as same as to the downward direction. This occurrence shows the inelastic buckling behavior with including the effect of little initial deformation. Whereas the beam size of ECB450x150x1696, which is in plastic range, the range that lateral torsional buckling does not apply, has the displacement hardly moving to the lateral side. Such behavior effect the beam strength encounter the full yielding of section mainly and directly affect to more difference of beam capacity while the Elliptical openings being present (as seen in Fig. 10(c)). The lateral deflection is found to be relatively small for the beam having low slenderness section, such as the beam ECB300x200x2955, which almost does not deflect laterally at the ultimate loading. The observation on stiffness of the beams also shows that there is almost no difference for the case of the plain web comparing with the Elliptical Cellular beams for the elastic behavior, but somewhat difference for the plastic behavior.

Table 2.   Example results of finite element analysis compare with AISC 360-10 standard.

Sections	L	AISC (consider at web-post section)				FEA	Reduction
		I_y	J	C_w	M		
	(mm)	(mm⁴)	(mm⁴)	(mm⁶)	(kN-m)	(kN-m)	(%)
ECB150x50	565	1.47×10^5	17079	7.53×10^8	18.04*	15.76	12.64
ECB150x50	997				15.92*	14.44	9.31
ECB150x50	1429				13.81*	13.05	5.49
ECB150x50	1861				11.69*	10.56	9.64
ECB150x50	2294				9.38	8.79	6.32
ECB150x50	2726				7.69	7.20	6.36
ECB150x50	3158				6.52	6.11	6.26
ECB150x50	3590				5.67	5.32	6.19
ECB450x150	1696	5.07×10^6	1.12×10^5	2.47×10^{11}	218.83*	190.90	12.76
ECB450x150	2992				178.14*	167.92	5.74
ECB450x150	4288				137.45*	133.90	2.59
ECB450x150	5584				88.19	87.51	0.77
ECB450x150	6881				63.22	62.43	1.25
ECB450x150	8177				48.81	47.95	1.76
ECB450x150	9473				39.60	38.73	2.19
ECB450x150	10769				33.28	32.44	2.54
ECB300x200	1168	1.60×10^7	2.78×10^5	3.32×10^{11}	210.89**	198.42	5.91
ECB300x200	2955				202.35*	193.31	4.47
ECB300x200	4743				175.81*	180.58	3.53
ECB300x200	6530				149.28*	154.30	3.22
ECB300x200	8318				118.57	116.59	1.67
ECB300x200	10106				92.63	91.13	1.62
ECB300x200	11893				76.15	75.01	1.49
ECB300x200	13681				64.74	63.86	1.36
ECB600x200	2261	1.74×10^7	3.91×10^5	1.50×10^{12}	537.16*	474.71	11.63
ECB600x200	3989				441.60*	403.52	8.62
ECB600x200	5717				346.04*	332.80	3.83
ECB600x200	7445				226.75	222.94	1.68
ECB600x200	9173				162.74	159.75	1.84
ECB600x200	10901				125.73	123.12	2.07
ECB600x200	12629				102.08	99.78	2.25
ECB600x200	14357				85.84	83.79	2.39
ECB900x200	2094	1.92×10^9	1.04×10^6	4.44×10^{12}	1232.30*	1050.00	14.79
ECB900x200	3391				1046.44*	935.56	10.60
ECB900x200	4686				860.63*	814.68	5.34
ECB900x200	5983				637.76	613.03	3.88
ECB900x200	8575				348.48	338.30	2.92
ECB900x200	11167				232.44	224.76	3.30
ECB900x200	13759				172.96	166.43	3.78
ECB900x200	16351				137.54	131.81	4.17
ECB900x200	18943				114.24	109.16	4.45
ECB900x200	21535				97.79	93.25	4.65

Lateral torsional buckling moment in inelastic range.
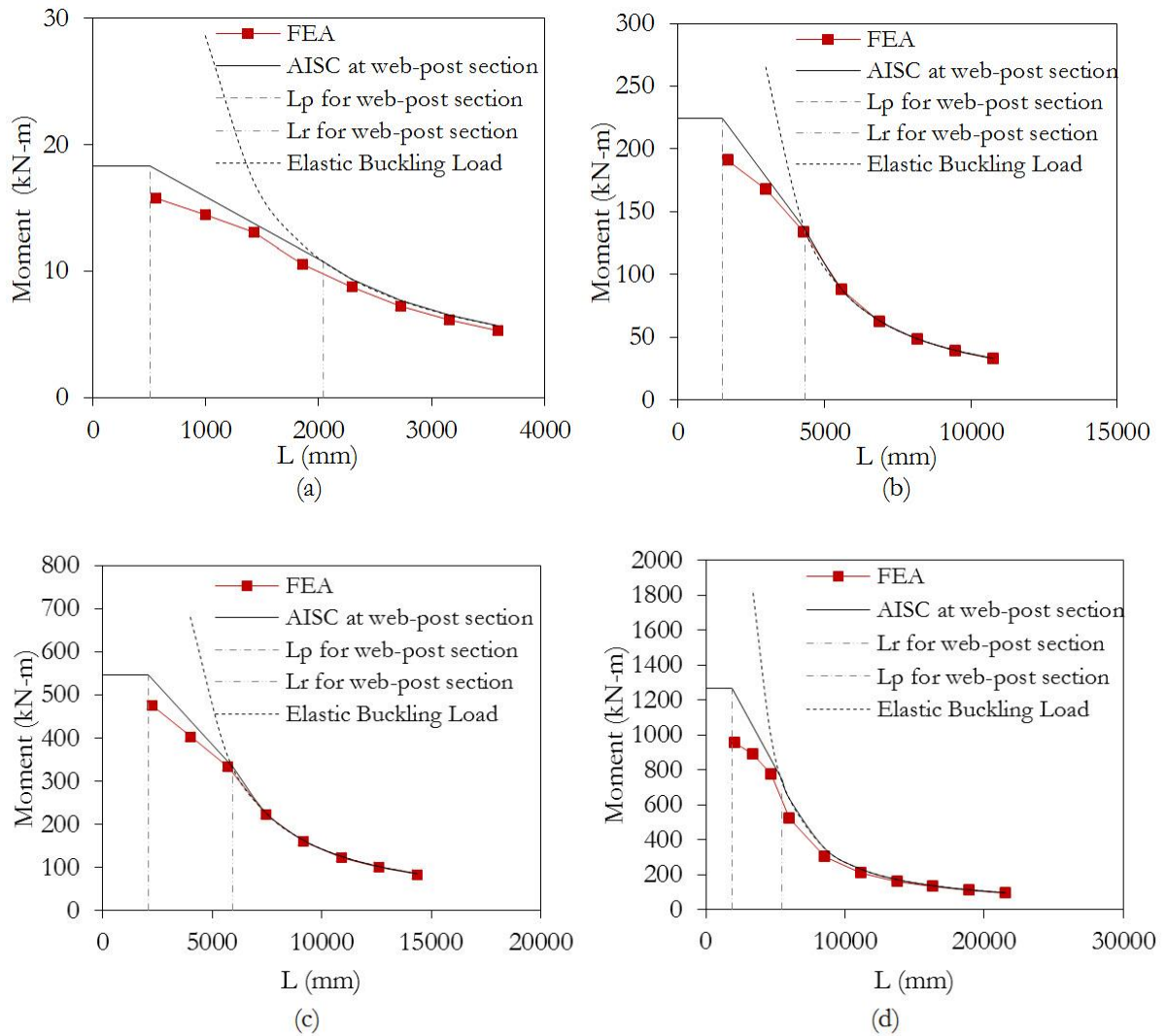**Plastic moment in plastic range.*

Fig. 7. Results of finite element analysis: (a) ECB150x50, (b) ECB450x150, (c) ECB600x200, (d) ECB900x200
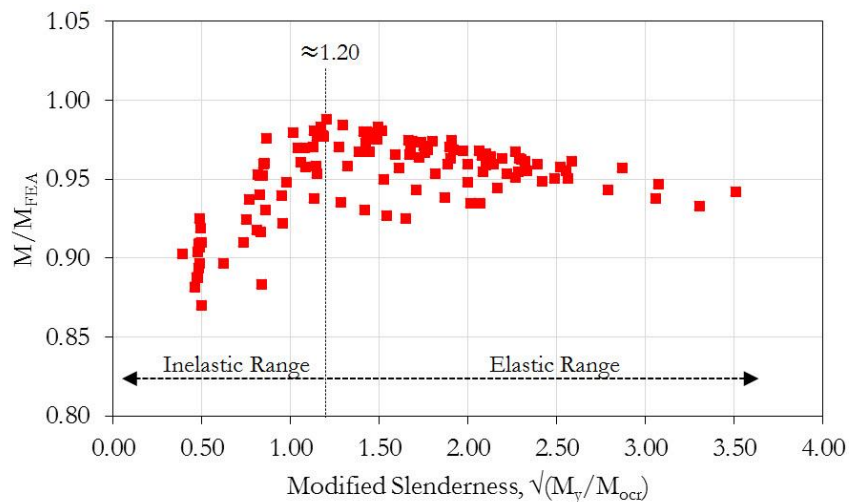

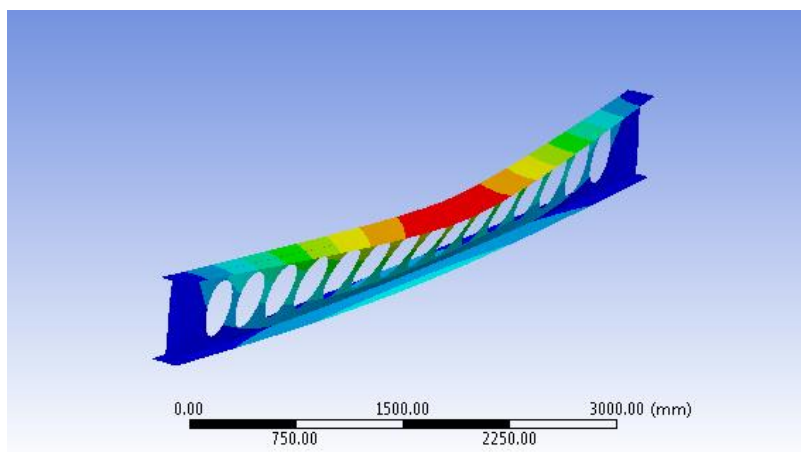Fig. 8. Graph of reduction factor against modified slenderness with various flange slenderness.

Fig. 9.   Result of finite element analysis in lateral torsional buckling of Elliptical Cellular beam.
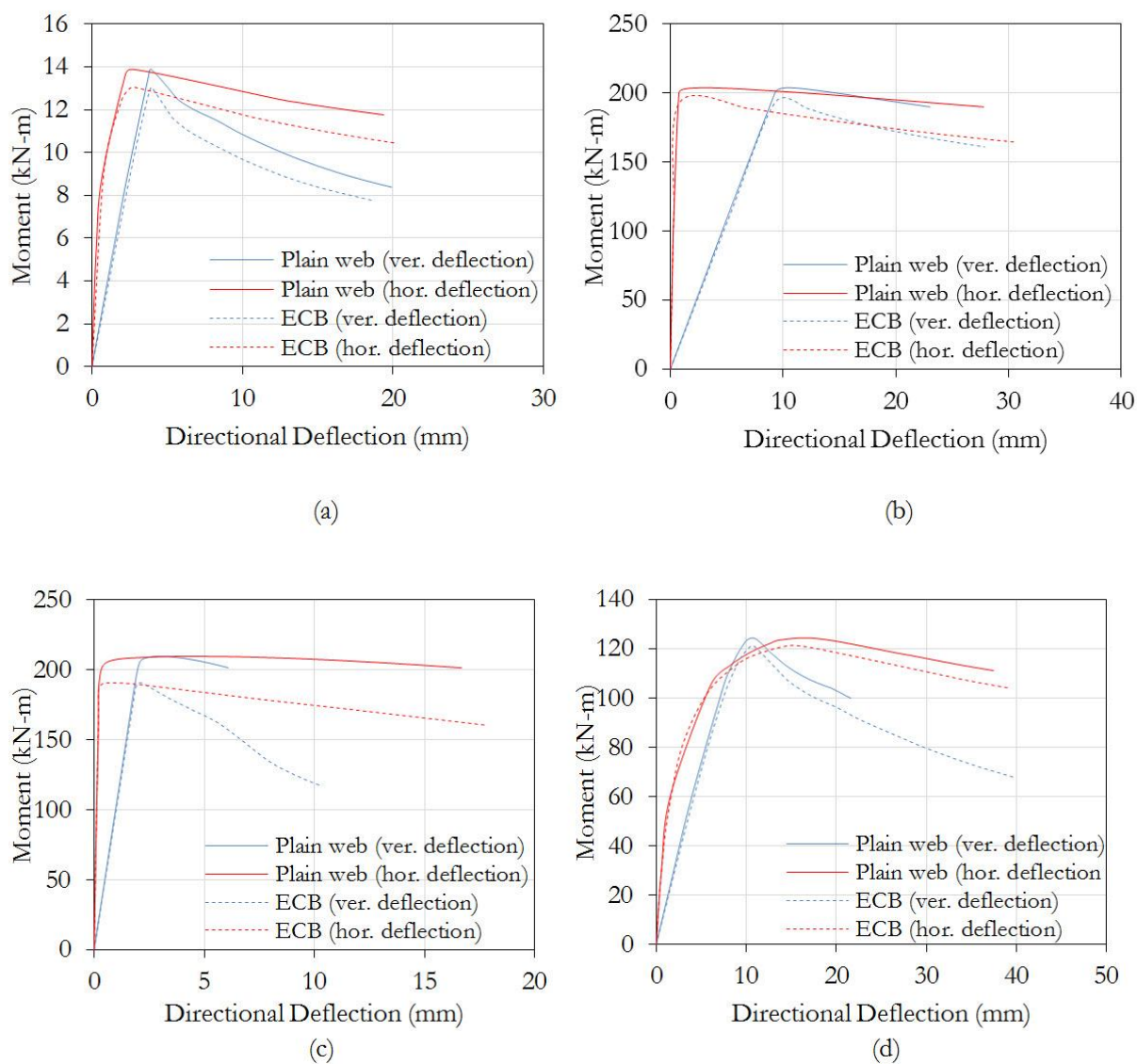

(a)

(b)

(c)

(d)

Fig. 10. Load-deflection   comparison   graph: (a)   ECB150x50x1429,   (b)   ECB300x200x2955,   (c) ECB450x150x1696, and (d) ECB450x150x4288.

6. Coefficient Function for Evaluate Lateral Torsional Buckling Moment of Elliptical Cellular Beams

As described in the earlier section that the equations to account for the lateral torsional buckling capacity used for the general steel beam are improper to represent the behavior for the Elliptical Cellular beam. Furthermore, using sectional properties at the web-opening section to design the Elliptical Cellular beam is unfairly conservative. Therefore, it is one of the main objectives for this paper is to propose applicable equation that developed from coefficient function obtained from the parametric study of the nonlinear buckling finite element analysis for more precise evaluation of the beam capacity. The proposed concept is to reduce the moment capacity from the standard equations using the sectional properties at the web-post section with a reduction factor, R_{ECB}. The scheme of the proposed concept has been plotted as shown in Fig. 11. The equations are illustrated in three ranges. The Elliptical Cellular beam that buckle in elastic range, the lateral torsional buckling moment capacity can be evaluated by using Eq. (5) that has a proposed reduction factor, R_{ECB}. This reduction factor is provided by empirical formula based on finite element results that the formula is depending on the sectional yielding moment at the web-post section, M_y, the critical lateral torsional buckling moment at the web-post section, M_{ocr}, and the slenderness of the top flange element, $b/2t_f$. For the inelastic range, Eq. (4) is developed based on the interpolation between the plastic moment at L_p and the elastic moment at L_r with the use of the reduction factor. Where in the equation, $M_{p,hole}$ is the plastic moment at the web-opening section, C_b is the lateral torsional buckling modification factor for nonuniform moment diagrams, I_y is the moment of inertia about minor axis, J is the torsional rigidity, C_w is the warping constant, S_x is the elastic section modulus taken about the x-axis, L_b is the length between lateral supports, L_p is the limiting length between plastic and inelastic behavior, L_r is the limiting length between inelastic and elastic behavior, F_y is yield stress of material, G is the shear modulus of elasticity, and E is the Young's modulus of elasticity. For the plastic range, the evaluation of Elliptical Cellular beam is necessary to use Eq. (3) that consider the sectional properties at the web-opening section because the beam will not undergo into the buckling behavior that affect the beam failure by fully yielding at web-opening section. It is noted that, for the elastic range, the main reason accounting for sectional properties at the web-post section is to simplifier the evaluation, the calculations of I_y, J and C_w.

In plastic length:

$$M_n = M_{p,hole} \tag{3}$$

In inelastic length:

$$M_n = C_b\left[M_{p,hole} - \left(M_{p,hole} - 0.7 \cdot R_{ECB} \cdot F_y S_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_{p,hole} \tag{4}$$

In elastic length:

$$M_n = R_{ECB} \cdot M_{cr} = R_{ECB} \cdot C_b \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w} \tag{5}$$

where

$$R_{ECB} = 0.9429 - 0.0135\sqrt{\frac{M_y}{M_{ocr}}} + 0.0063\left(\frac{b}{2t_f}\right)$$
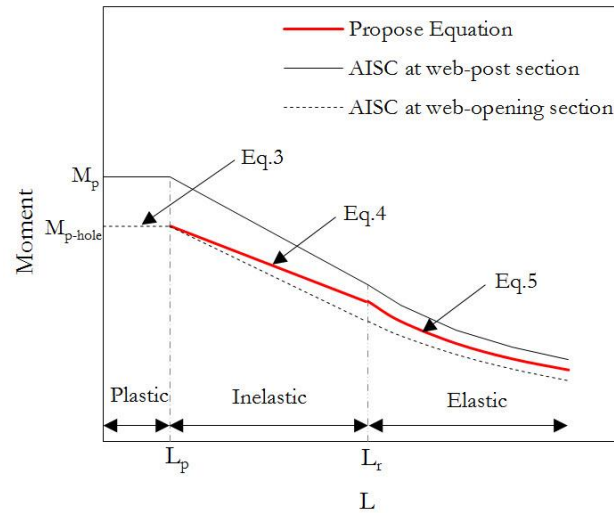
Fig. 11. Schematic plot for evaluating lateral torsional moment of Elliptical Cellular beam.
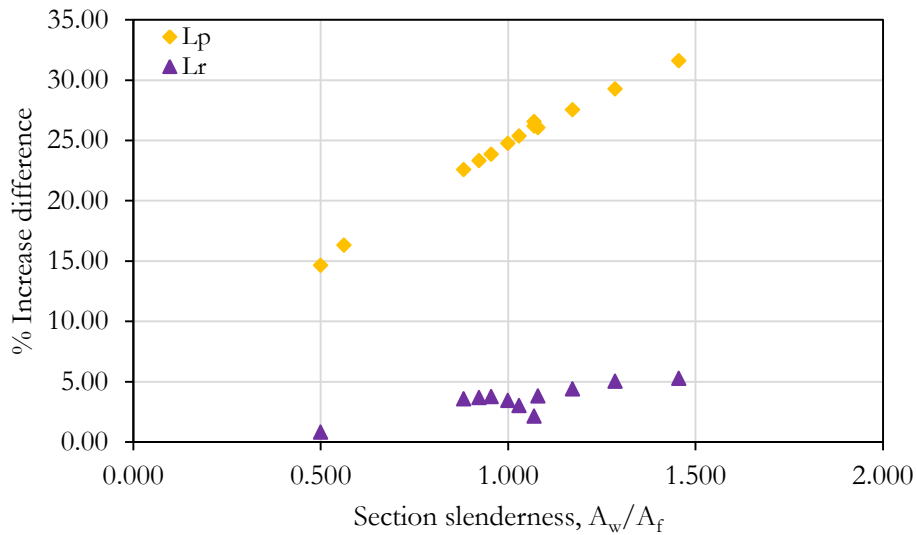

Fig. 12. Difference when considering the sectional properties at web-post section and at web-opening section that effect the increasing values of L_p and L_r.
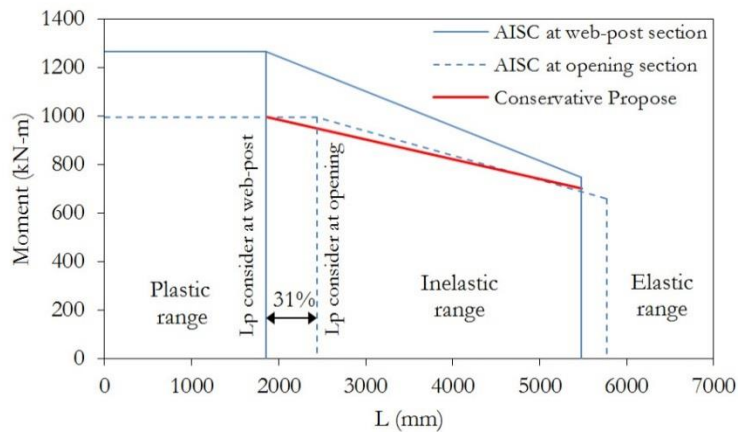

Fig. 13. Conservative propose for L_p (example results of ECB900x200 specimen).

In consideration the values of L_p and L_r, the results of the study show that there are negligible difference of L_r between consideration the sectional properties at the web-post section and at the web-opening section. The difference is about 5 percentage when comparing the section slenderness, A_w/A_f, as shown in Fig. 12. However, in the consideration of L_p, it is found that there are noticeable difference of approximately 15 percentage to up to 35 percentage with the variation of the section slenderness, A_w/A_f. Fig. 13 is plotted to further demonstrate in detail of an example case, ECB900x200, which is the case of the extreme L_p difference. It is observed that using the sectional properties at the web-post section is convenient and yield slightly conservative results. Therefore, in the procedure of calculation, it is also appropriate to propose to use the sectional properties to determine L_p and L_r at the web-post section.

The results of the proposed procedure in evaluation the lateral torsional buckling moment capacity of Elliptical Cellular beams are compared with the results of the calculation conformed to the AISC standard with both considering sectional properties at the web-post section and at the web-opening section (as seen in Fig. 14). The comparison shows that the results of AISC Standard eqution using sectional properties at the web-post section obviously give unconservative results. The unconservative result is about 5 percent in elastic buckling range and when the beam lengths are in inelastic buckling range, the unconservative can increasingly be up to 20 percent with the increasing effect in the plastic range. Considering AISC equation using sectional properties at the web-opening section, it can be found that the results are always below the conservative line for up to 10 percent. The use of sectional properties at the web-opening section also shows the wide distributed ranges of the results. Finally, focusing on the results evaluated by using the reduction factor from this proposal, it is observed that the results have given reasonable conservative evaluation, more precision, and less distribution of the results than the two previous calculation methods.
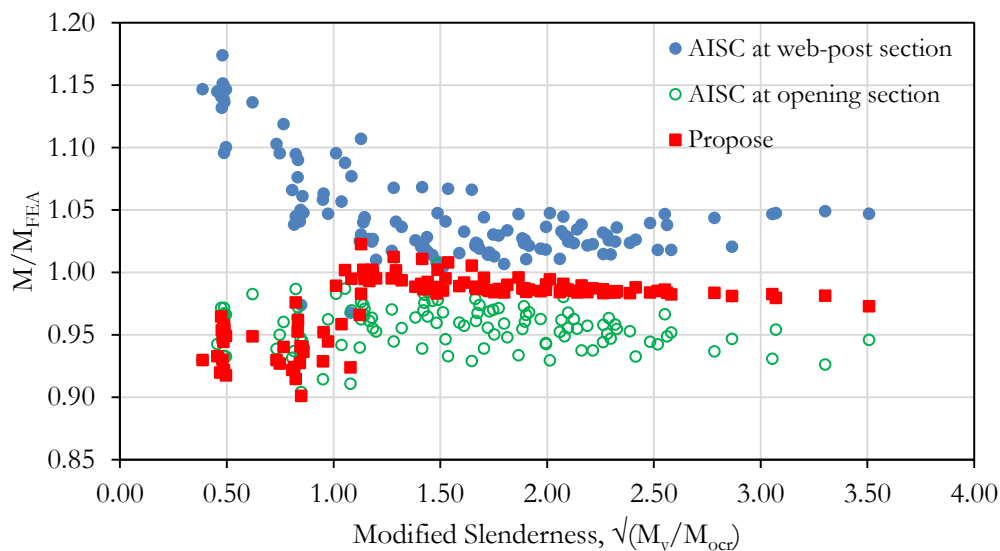

Fig. 14. Result of the comparison of lateral torsional buckling moment evaluation of Elliptical Cellular beam.
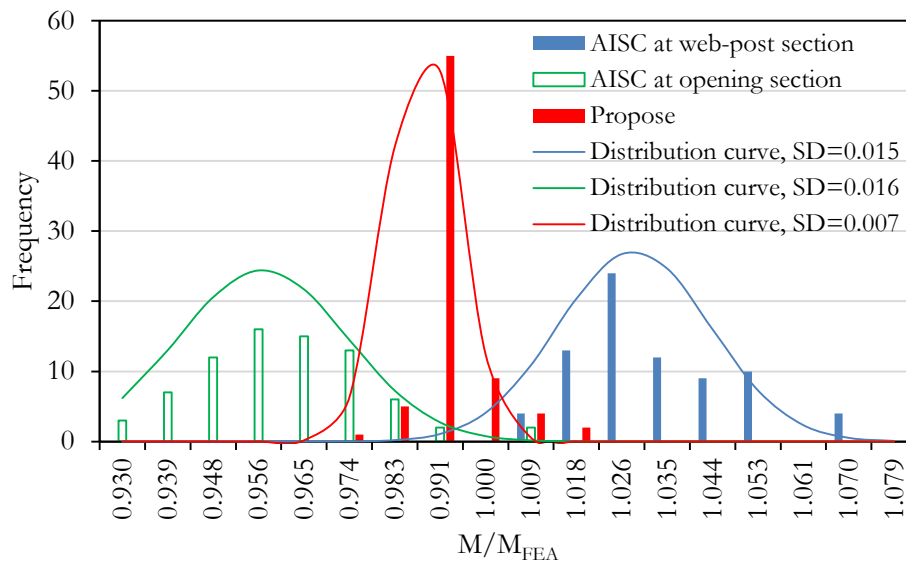
Fig. 15. Probability distribution graph of moment capacity evaluation.

Furthermore, considering statistics analysis, the probability distribution of all data in elastic buckling range is plotted in Fig. 15. The calculation of lateral torsional buckling moment of Elliptical Cellular beams that follow the AISC standard with the use of sectional properties at the web-post section yields the results with the arithmetic mean of 1.03 and the distribution in standard deviation (S.D.) of 0.0147. The calculation with the use of sectional properties at the web-post section gives the arithmetic mean of 0.96 and the standard deviation of 0.0163. Lastly, the evaluation using of the reduction factor, R_{ECB}, concept from this proposal provides precise results with the arithmetic mean of 0.99 and the less distribution of results with the standard deviation of 0.0068.

In addition, from the study of the lateral torsional buckling moment of Elliptical Cellular beams in other load cases, it is found that the presence of elliptical openings along the beam web has no effect directly to the lateral torsional buckling moment, especially, in elastic buckling behavior range. But in the inelastic and plastic range, it was found that some of the beam models had encoutnered another failure modes such as web-post buckling, web-post shear failure in the region of high shear force, or Vierendeel mechanism failure, similarly to the previous study by P. Panedpojaman, W. Sae-Long and T. Chub-uppakarn [23]. Likewise the use of the method to reduce the moment gradient factor, C_b, in the inelastic range for other load cases recommended by A. Mohebkhah [15].

7. Conclusion

The linear and nonlinear buckling analysis by using finite element method have been performed in this study to investigate the lateral torsional buckling moment capacity of Elliptical Cellular beams in both elastic and inelastic buckling range. The main essences of this study can be concluded as followings:
1. In the evaluation of the lateral torsional buckling moment of Elliptical Cellular beams, by considering section properties at the web-post section, the equation given in AISC standard is obviously unsafe. The results of this unconservative design comparing to the finite element analysis show that the unconservative is about 5 percent in elastic range, and significantly up to 20 percent in inelastic and plastic range.
2. The lateral torsional buckling moment of Elliptical Cellular beams can be conservatively design by using the reduction factor from this proposal, R_{ECB}. The results using the proposed reduction factor (R_{ECB}) to evaluate the lateral torsional buckling moment capacity show appropriate and precise results for design with more accuracy, more economical and less distribution of results.
3. The accounting for sectional properties at the web-post section and the web-opening section of Elliptical Cellular beams yield negligible difference in the value of L_r. However, for the value of L_p, it can be different for up to 31 percent. It is slightly more conservative to calculate the value of L_p by

using sectional properties at the web-post section. Therefore, this study propose to use sectional properties of Elliptical Cellular beams at the web-post section in both L_p and L_r.

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